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ANALYSIS OF LARGE URBAN FIRES

By

R. D. Small
D. A. Larson
H. L. Brode

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Large urban fires have resulted from natural disasters, explosions, and wartime actions. A single-megaton nuclear weapon can ignite hundreds of square kilometers. This report formulates a model that identifies several separate regions or components of such large area fires, and analyzes them individually. The conservation equations for a compressible flow are reduced, using asymptotic analysis, to simplified equation sets appropriate for each component region. Expansions are constructed using the burning-zone aspect ratio as the small parameter. The ongoing combustion processes are modeled by a spatially

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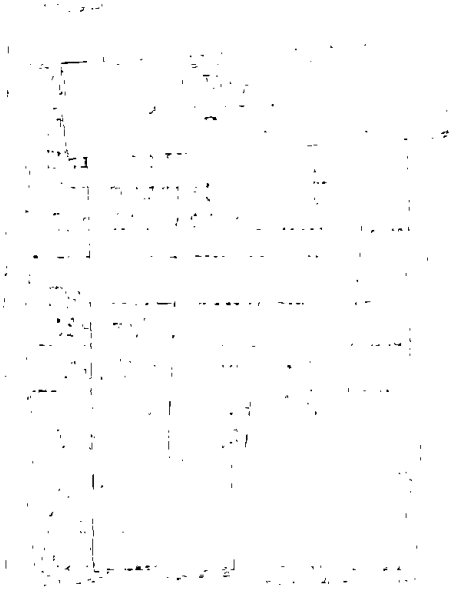
dependent volume heat source. The theory of large area fires thus developed is applicable to arbitrarily large fires and, in the lower limit, to scale fires the size of the multiple-fuel-bed Flambeau experiments.

The boundary value problem and solution algorithm for computation of the velocities and thermodynamic variables in and around the burning zone are detailed. Numerical results are given for the nominal case of a 10 km fire. The influence of radiation, turbulent diffusion of heat and momentum, and heat addition rate are explored and analyzed. The results show a large-velocity horizontal inflow being turned in the burning (heat addition) region to a vertical column flow, the convection column having a width comparable to that of the fire. The numerical results reveal the nonlinear interchanges of momentum and energy in the burning region and the early plume. An experimental multiple-fuel-bed Flambeau fire is also analyzed. For that case, the equations describing the convection column are shown to reduce to the classical plume equations. Numerical results are consistent with experimental observations, and resemble those obtained for the 10 km fire.

PREFACE

This report continues Pacific-Sierra Research Corporation's model development and analysis of large urban fires. Analysis and results are presented for a nominal 10 km area fire and a multiple-fuel-bed Flambeau fire. Portions of this report will be submitted for presentation and publication to the Nineteenth International Symposium on Combustion, sponsored by The Combustion Institute.



EXECUTIVE SUMMARY

Large urban fires have resulted from natural disasters, explosions, and wartime actions. In many cases, entire urban areas were totally destroyed despite firefighters' efforts to contain the flames. The World War II firebombing raids on European and Japanese population centers caused immense damage and hundreds of thousands of casualties. Several ignited firestorms, with hurricane-force winds, high street-level temperatures, high concentrations of carbon monoxide, and complete burning of all combustible materials within the firestorm boundaries. Firestorms also produced a high number of casualties, seldom ameliorated even by concerted rescue efforts.

Large urban fires are a much greater threat in the age of nuclear weapons than ever before. Hundreds of square kilometers of an urban (or wildland) area can be ignited simultaneously by a single-megaton nuclear weapon. Indeed, superfires of unprecedented size could dwarf the tremendous fires of World War II.

After the firestorm in Hamburg, civil defense leaders in Germany addressed the problem of protecting the cities and the civilian population from such fierce fires. On the basis of the meager technical information then available, guidelines for rescue operations were outlined. However, the physical characteristics of the firestorm environment had not yet been defined--e.g., winds, temperatures, gas concentrations, sizes of combustible areas, and populations at risk. Such information is essential if planners are to design procedures for fire suppression/containment and population protection/rescue.

Today, years later, we are still without a complete and accurate picture of the superfire environment. No analysis has ever focused on the nature of the flow in and around the fire itself--which is the most vital information. Most studies have simply attempted to extrapolate results from classical plume theories, known to be valid only well above a small free-burning fire.

This report uses a new analytical approach to analyze the superfire environment. Our approach is based on a "component" model in

which different sets of equations are employed to represent the fundamentally different physics in separate parts of the fire-generated flow. Asymptotic analysis is used to derive the individual equation sets from standard conservation laws.

The "component" approach allows us to consider in detail the nature of the superfire environment in the critical region in and around the fire itself. Our analysis of that "combustion" region constitutes the first--and most interesting--model component.

The intensity and extent of the fires in the combustion region largely determine the characteristics of the entire flow field. Cold air is drawn into the burning zone and heated by the ongoing combustion processes. The heated air rises buoyantly, establishing a natural convection column. The buoyant air in the column cools and expands until a local equilibrium is established either in the upper atmosphere or at an inversion layer (of sufficient magnitude). Finally, a mesoscale recirculation pattern is established similar to a nocturnal urban heat island flow. Analyses of the convection column and recirculation flows produce the other basic model components.

We focus on the combustion region. Our analysis is valid in the upper limit for large urban fires with horizontal dimensions on the order of tens of kilometers and in the lower limit for fires the size of the multiple-fuel-bed Flambeau fires.

The first predictions concern the environment for a nominal large urban fire (with a 10 km radius). The sensitivity of the predictions to changes in model parameters is then considered. An experimental Flambeau fire (of radius $\sim$0.25 km) is also analyzed. The results show reasonable agreement in all parameters when inflow horizontal velocities are selected to correspond to those observed.

Numerical simulation also shows that the type of plume observed over the Flambeau fire (i.e., with a width comparable to that of the fire) can be established only if turbulent mixing plays a major role in the combustion region dynamics. We anticipate that turbulence will play such a role in large urban fires.

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I. INTRODUCTION

Each year, fires cause a tremendous loss of life and property in the United States. The spread of most urban structure fires is readily controlled by local firefighters, although there have been several notable exceptions [Lowe, 1979; Brode, 1980]. However, simultaneous fires such as those caused by natural and other disasters--e.g., earthquakes, explosions--often overwhelm local firefighters and destroy entire urban tracts. The extensive acreages annually burned by wildland fires would be even larger without coordinated action by great numbers of firefighters [Emmons, 1963, 1978].

An ordinary urban fire usually involves relatively few structures. In wartime, however, fires are deliberately set with the intention of burning whole cities. Multiple fires in large urban areas have proven extremely difficult to combat. Indeed, some have become superfires governed by phenomena not present in small urban fires.

The incendiary raids in Europe and Japan during World War II set multiple fires over fairly large urban tracts [Bond, 1951; Irving, 1963]. Several raids with highly concentrated bombing ignited firestorms that totally destroyed entire city sections (Hamburg, Dresden, Kassel, Darmstadt). Postwar studies based on survivor reports described fires simultaneously burning over tens of square kilometers (Hiroshima, 12 km^2; Dresden, 21 to 28 km^2; Hamburg, 21 km^2) and consuming all combustible materials within their boundaries. Hurricane-force winds, high-level street temperatures, and combustion intensities never previously observed were reported. Those experiences demonstrate that large area fires engender phenomena radically different from those of smaller fires.

Modern nuclear weapons can set fires over far larger areas than affected by any previous warfare or natural disaster. The thermal reach of megaton-size weapons can cause fires over more than 100 km^2. For example, a one-megaton explosion (height of burst 700 m) can irradiate an area 7 km from ground zero with 30 cal/cm^2 of thermal

radiation. This is more than sufficient to ignite lightweight household materials and many typical exterior materials [Glasstone and Dolan, 1977]. The many thousands of simultaneous ignitions that will occur within seconds of the weapon burst over extensive urban areas could cause firestorms that would dwarf those that occurred in World War II.

This report presents a method for analyzing large area fires. A large area fire is herein defined as one with a flame height much smaller than its horizontal extent, and a convection column height comparable to a scale height of the atmosphere. Our analytical method should be applicable in the lower limit to fires the size of the multiple-fuel-bed *Flambeau* experiments [Countryman, 1969] and to nuclear-weapon-induced fires in the upper limit--as well as to city fires caused by natural disasters and wildland fires involving large areas.

TECHNICAL BACKGROUND

Most previous research concerning free-burning fires has been restricted to small fires and their plumes. There are significant phenomenological differences between small and large fires as well as between line and area fires [Lee, 1972; Small and Brode, 1980]. For example, the flow phenomena observed in small, free-burning fires can be related to local entrainment of ambient air [Cox and Chitty, 1980; McCaffrey, 1979], whereas additional mechanisms apparently are present in larger fires. This is evident from the large wind velocities observed in the World War II firestorms and in large wildland fires.

Similarly, the characteristics of line fires--such as occur in some wildlands or in urban fires (e.g., Tokyo, 1945)--are controlled by phenomena different from those of an area fire. A line fire is spread by local mechanisms such as conduction, convection, and radiation into the unburned regions [Emmons, 1963]. Area fires can be more nearly stationary and their behavior determined largely by integral properties--e.g., fire size, total heat release, and general atmospheric conditions.

Modeling of fires is extraordinarily complex, and a complete analysis would include description of both microscale and macroscale

phenomena. Indeed, many of the phenomena (e.g., chemistry and kinetics of fires, radiation, turbulence) that play a role in even a "simple" fire can still be described only qualitatively.

The plumes generated by free-burning fires have been the subject of several analyses. Similarity solutions [Morton, Taylor, and Turner, 1956; Morton, 1959; Yokoi, 1960] have been obtained using either an entrainment law or a mixing-length hypothesis. Such solutions are applicable for long, thin plumes and only well above the fire zone. Improvements to the basic plume theories have included piecewise integration of the solution through a real atmosphere [Murgai and Emmons, 1960], inclusion of radiation effects [Murgai, 1962; Smith, 1967], and consideration of the effects of combustion and variable gas composition [Nielsen and Tao, 1965].

The simple plume theories postulate relatively low inflow velocities at the edge of the convective plume and cannot be extrapolated to account for the higher velocity flows that occur in the fire zone. Even stoichiometric combustion in large, free-burning fires would require higher velocities than can be obtained through an edge effect such as entrainment. Simple scaling arguments [Byram, 1966; Small and Brode, 1980] suggest that the inflow velocities depend on the fire intensity and size.

The computations of Smith, Morton, and Leslie [1975] provide a first treatment of fire-generated winds near the fire zone as well as above it. Their results indicate a strong coupling of dynamic forces with the induced fire wind, even though density changes are assumed small (Boussinesq approximation) and the combustion processes are represented only by a heat addition rate at the ground boundary.

Our analysis provides detailed treatment of a finite-volume combustion zone and resolution of the dynamics and thermodynamics in and around the fire.

SUMMARY OF ANALYSIS

Our analytical approach is to consider the special features of each region (combustion zone, convection column, and far field of a superfire) and to use asymptotic expansions for formulating the appropriate equation sets. Scaled equations of mass, momentum, and energy

conservation, and an equation of state are introduced for each region, and an overall description of the flow is obtained by matching at the boundaries of those separate regions.

The basic features of the entire flow field are determined by the interactions in the combustion zone. Although that region is considerably smaller than the others (the convection column and the mesoscale atmospheric recirculation regions), the accurate resolution of its dynamics and thermodynamics are of primary importance in the modeling. The use of a component model facilitates such resolution and represents a significant departure from previous fire research [Morton, Taylor, and Turner, 1956; Murgai and Emmons, 1960; Nielsen and Tao, 1965; Smith, Morton, and Leslie, 1975].

Accordingly, we have emphasized the analysis of the combustion zone in our theoretical and numerical development. The heat release generated by the fires is modeled by a spatially-dependent volume heat source. Since density and temperature changes can be large, the conservation equations for a compressible gas are applied without the Boussinesq approximation. This allows coupling of the fire heat release, buoyancy, and pressure fields. Our analysis shows that several solutions can be obtained consistent with either a "thin" plume or a "thick" plume. Since large, free-burning fires usually engender convection column widths comparable to the horizontal fuel-bed dimension, we emphasize the "thick" column solution.

The theory is applied to a 10 km urban fire and one Flambeau experimental fire. The computed results detail the dynamics and thermodynamics of the combustion zone and clearly show the turning of a horizontal inflow velocity by the heat release and strong buoyancy into a vertical column flow. The calculation is carried vertically to approximately ten flame heights, and the results prescribe the initial conditions for the convective plume flow.

II. MODELING AND ANALYSIS

Our analysis considers an urban area covered by fire; the burning region need not be uniform. Such fires have resulted from natural disasters, wartime fire bombing, and the detonation of a nuclear weapon. The hot combustion products rise in the atmosphere and form a convection column above the burning region. The column height depends on conditions in the atmosphere as well as in the burning region. The initial temperature and momentum imparted to the gas in the burning region and the density difference between the hot column and the surrounding atmosphere all influence the height and character of the smoke column. The gases eventually establish equilibrium with the local atmosphere and spread laterally at the top of the column. A mesoscale far-field recirculation develops, similar to urban heat island flows [Delage and Taylor, 1970]. Figure 1 illustrates the basic features of a large area fire.

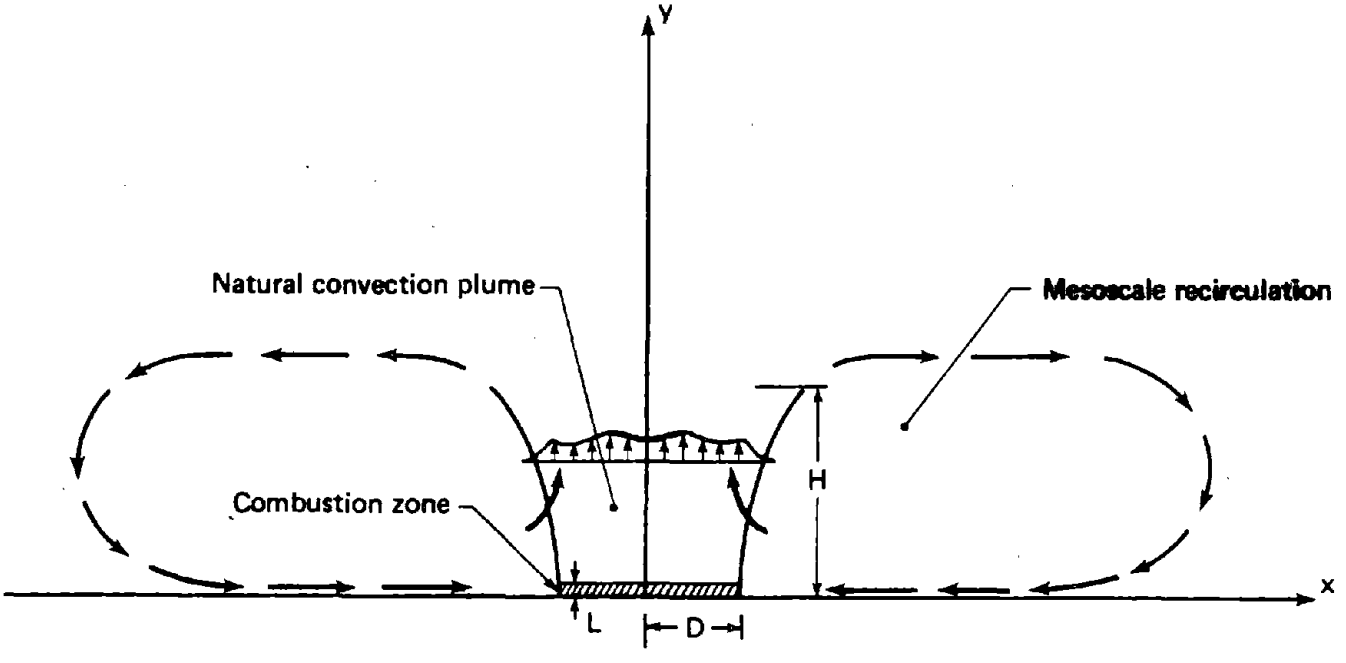


Fig. 1--Schematic of large area fire

The combustion zone is characterized by a heating zone of height L and half-width D. Considering a large area fire, then, the combustion-zone aspect ratio (L/D) forms a small parameter

$$\varepsilon = L/D \ ,$$

which approaches zero in the limit of an infinite fire. The half-width D can be of the same order as the convection column height H, implying a convection-column aspect ratio (D/H) of order unity.

PHYSICS

The dominant physical effect in the combustion zone is the heat generated by the fire. Heated by the ongoing combustion, air becomes buoyant and rises, carrying with it smoke particulates and other by-products of combustion. The resulting flow is turbulent, and experiences radiation losses.

The complex combustion mechanisms are not considered in detail here. We model the overall effect of the burning by using a spatially dependent volumetric heat addition rate in the combustion zone. It is tacitly assumed that the buoyancy generated by the burning processes is reproduced by this volume addition. A more detailed description of the combustion physics may be introduced as a model refinement in a subsequent report.

In and around the combustion zone, the mean fire-generated flow is taken to be that of an ideal, compressible gas. Since that flow is expected to experience large changes in temperature but only small changes in pressure, the Boussinesq approximation is not employed. We take into account the effects of gas radiation, because experimental [McCaffrey, 1979] and analytical [Murgai, 1962; Smith, 1967] work suggests that radiation plays a significant role in reducing local air temperatures to nearly atmospheric values within a few flame heights above the fire. For simplicity, radiation losses are now modeled as being grey body, or "transparent" [Murgai, 1962]. Finally, analysis also shows that turbulent (Reynolds) stresses have an

important effect at least in the neighborhood of the symmetry axis. Standard eddy diffusivities are used in modeling the stresses. More detailed descriptions of the radiation and turbulence effects may be introduced later.

In the natural convection plume, hot, buoyant air from the combustion zone rises, mixes, spreads, and finally spills out laterally as it passes an equilibrium level in the upper atmosphere. The characteristics of the column depend critically on its aspect ratio D/H. Significant mixing occurs across long, narrow, Flambeau-type plumes, and the plume width at the top is greater than at the bottom. For larger urban fire plumes, which are as wide as they are tall, entrainment and mixing occur principally very near the sides of the plume and the flow is basically one-dimensional, with only a slight spreading along the sides. Figure 2 illustrates model components of a large urban fire.

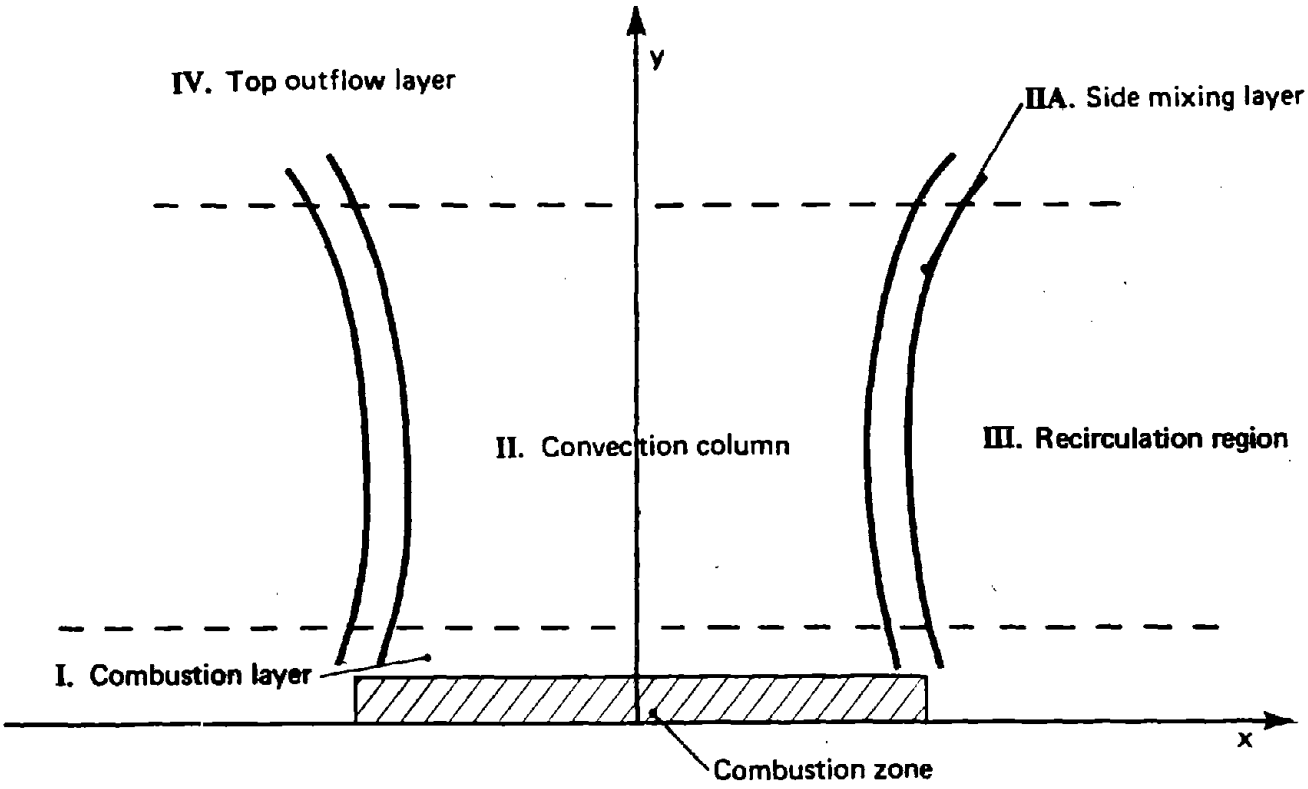


Fig. 2--Components of large urban fire model

BASIC EQUATIONS AND SCALING

The conservation equations for mass, momentum, and energy, and an equation of state appropriate for the steady-state description of a two-dimensional, large area fire are as follows:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \ ;$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mathcal{E}_{11}\frac{\partial^2 u}{\partial x^2} + \mathcal{E}_{12}\frac{\partial^2 u}{\partial y^2} \ ;$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\left(\frac{\partial P}{\partial y} + g\rho\right) + \mathcal{E}_{21}\frac{\partial^2 v}{\partial x^2} + \mathcal{E}_{22}\frac{\partial^2 v}{\partial y^2} \ ;$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + Q \cdot q(x, y) - (4\pi\hat{\sigma}k^*)[T^4 - T_\infty^4(y)]$$

$$+ k_1\frac{\partial^2 T}{\partial x^2} + k_2\frac{\partial^2 T}{\partial y^2} \ ;$$

$$P = \rho RT \ . \qquad (2.1)$$

The diffusion coefficients $\mathcal{E}_{ij}$ and k_i represent all dissipative processes (both molecular and turbulent) for momentum and heat. The Reynolds stresses are assumed to be proportional to appropriate second-derivative terms. The term $Q \cdot q(x, y)$ is the heat addition rate due to combustion, with Q representing the mean rate and $q(x, y)$ a specified spatial distribution. The radiation loss is modeled by $(4\pi\hat{\sigma}k^*)[T^4 - \hat{T}_\infty^4(y)]$ [Murgai, 1962], where $(k^*)^{-1}$ and $\hat{\sigma}$ are the emission mean free path of the radiation and the Stefan constant, respectively, and $\hat{T}_\infty(y)$ is the far-field ambient temperature.

Pressure (P), density (ρ), and temperature (T) are expected to be of the same order of magnitude in all regions of interest as they are in the far-field atmosphere. Ground-level atmospheric values hence serve as nominal scales for these variables. Since the driving force for the entire flow field lies in the combustion zone, the characteristic dimensions and flow speeds of that zone are used

as nominal scales (denoted by braces) for spatial coordinates and velocities, as follows:

$$\{x\} = D,\ \{y\} = L\ ; \qquad \{P\} = P_a,\ \{\rho\} = \rho_a,\ \{T\} = T_a\ ;$$

$$\{u\} = U,\ \{v\} = \varepsilon U, \qquad U \text{ yet to be chosen .} \tag{2.2}$$

The subscript a refers to ground-level atmospheric values, ε is the combustion-zone aspect ratio ($\varepsilon << 1$), and U is to be chosen so as to balance the terms for convective transport and heat addition in the energy equation (i.e., so that the equation represents a flow driven by combustive heating). The scaling for u and v is chosen to preserve the two-dimensional structure of the continuity equation [Eq. (2.1)], subject to the x and y scalings.

The nominal scalings in Eq. (2.2) are appropriate for the study of region I (Fig. 2). Other regions require rescalings. For example, for $H \sim D$ and $\varepsilon = L/D$, the appropriate scaling for y in regions II and III is $\{y\} = H$, in contrast to Eq. (2.2). All rescalings are discussed as needed in our analysis.

The nondimensional version of Eq. (2.1), obtained by scaling Eqs. (2.1) using Eqs. (2.2), is

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0\ ;$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\left(\frac{1}{\delta_1}\right)\frac{\partial P}{\partial x} + M_{11}\frac{\partial^2 u}{\partial x^2} + M_{12}\frac{\partial^2 u}{\partial y^2}\ ;$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\left(\frac{1}{\varepsilon^2\delta_1}\right)\frac{\partial P}{\partial y} - \left(\frac{1}{\varepsilon\delta_2}\right)\rho + M_{21}\frac{\partial^2 v}{\partial x^2} + M_{22}\frac{\partial^2 v}{\partial y^2}\ ;$$

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + \beta\{q(x, y) - \sigma[T^4 - T_\infty^4(y)]\}$$

$$+ K_1\frac{\partial^2 T}{\partial x^2} + K_2\frac{\partial^2 T}{\partial y^2}\ ;$$

$$P = \rho T\ ; \tag{2.3}$$

where

$$\delta_1 = U^2/P_a/\rho_a \; ; \qquad \delta_2 = U^2/gD \; ;$$

$$M_{1j} = \varepsilon\mathcal{E}_{1j}/\rho_a UL \quad \text{for } j = 1, 2 \; ; \qquad K_1 = \varepsilon k_1/\rho_a UL \; ;$$

$$M_{2j} = \mathcal{E}_{2j}/\varepsilon\rho_a UL \quad \text{for } j = 1, 2 \; ; \qquad K_2 = k_2/\varepsilon\rho_a UL \; ;$$

$$\sigma = 4\pi\hat{\sigma}k^* T_a^4/Q \; ; \qquad T_\infty(y) = \hat{T}_\infty(y)/T_a \; ;$$

$$\beta = (\gamma - 1)/\gamma \cdot QD/P_a U \; ; \qquad \gamma = 1.4 \; ; \tag{2.4}$$

and $q(x, y)$ can be varied, subject only to

$$q(x, y) \equiv 0 \quad \text{for} \quad y > 1 \quad \text{and/or} \quad |x| > 1 \; . \tag{2.5}$$

Setting $\beta = 1$, so that the convective terms in the energy equation [Eq. (2.3)] are balanced with the heat source term, we find the nominal horizontal velocity scale U [Eq. (2.2)] to be

$$U = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{QD}{P_a}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{QL}{P_a \varepsilon}\right) \; . \tag{2.6}$$

For $QL \sim 5.8 \times 10^{+4}$ cal/m^2-sec [*DCPA*, 1973], $L \sim 10^2$ m, and $D \sim 10^4$ m, then

$$U \sim 67 \; \frac{m}{sec} \tag{2.7}$$

and increases (or decreases) proportionately with QL and inversely with ε.

Using the values for U, L, D, P_a, ρ_a, and g, then from Eq. (2.4) δ_1, $\delta_2 \sim 0(\varepsilon)$. Equations (2.3) can thus be rewritten in final form as

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \ ;$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{B}{\varepsilon}\frac{\partial P}{\partial x} + M_{11}\frac{\partial^2 u}{\partial x^2} + M_{12}\frac{\partial^2 u}{\partial y^2} \ ;$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{B}{\varepsilon^3}\left(\frac{\partial P}{\partial y} + \varepsilon A\rho\right) + M_{21}\frac{\partial^2 v}{\partial x^2} + M_{22}\frac{\partial^2 v}{\partial y^2} \ ;$$

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\gamma - 1}{\gamma}\left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + q(x, y) - \sigma[T^4 - T_\infty^4(y)]$$

$$+ K_1\frac{\partial^2 T}{\partial x^2} + K_2\frac{\partial^2 T}{\partial y^2} \ ;$$

$$P = \rho T \ , \qquad (2.8)$$

where

$$A = \frac{\delta_1}{\delta_2} \sim 1 \ , \qquad B = \frac{\varepsilon}{\delta_1} \sim 0.16 \ ,$$

$$\sigma \sim 0.023 \quad (\text{for } \hat{\sigma} = 1.354 \times 10^{-11} \text{ kcal/m}^2\text{-sec-(°K)}^4 \ ,$$

$$k^{*-1} \sim 100 \text{ m}) \ . \qquad (2.9)$$

The coefficients M_{ij} and K_j, which characterize the turbulence effects, are left as parameters in this basic set of model equations. They are estimated as necessary in the analysis.

Most of our analysis and discussion concerns Eqs. (2.8). One section, however, presents a brief parallel analysis of a multiple-fuel-bed Flambeau fire. In this case, L and D differ significantly from the previous order estimates, so we consider a correspondingly revised version of Eq. (2.8).

ASYMPTOTIC ANALYSIS

Our component description of a large area fire involves asymptotic analysis in the limit where the combustion-zone aspect ratio ε (=L/D)

tends toward zero. Asymptotic expansions for the solution of the model equations in Eq. (2.8) are developed for each component region in Fig. 2 above, and the expansions are then suitably matched. The matching proceeds as diagrammed in Fig. 3.

The key asymptotic solution is that for region I, the combustion layer. Once it is developed, expansions for the convection column and side mixing layer (regions II and IIA, respectively) are found and

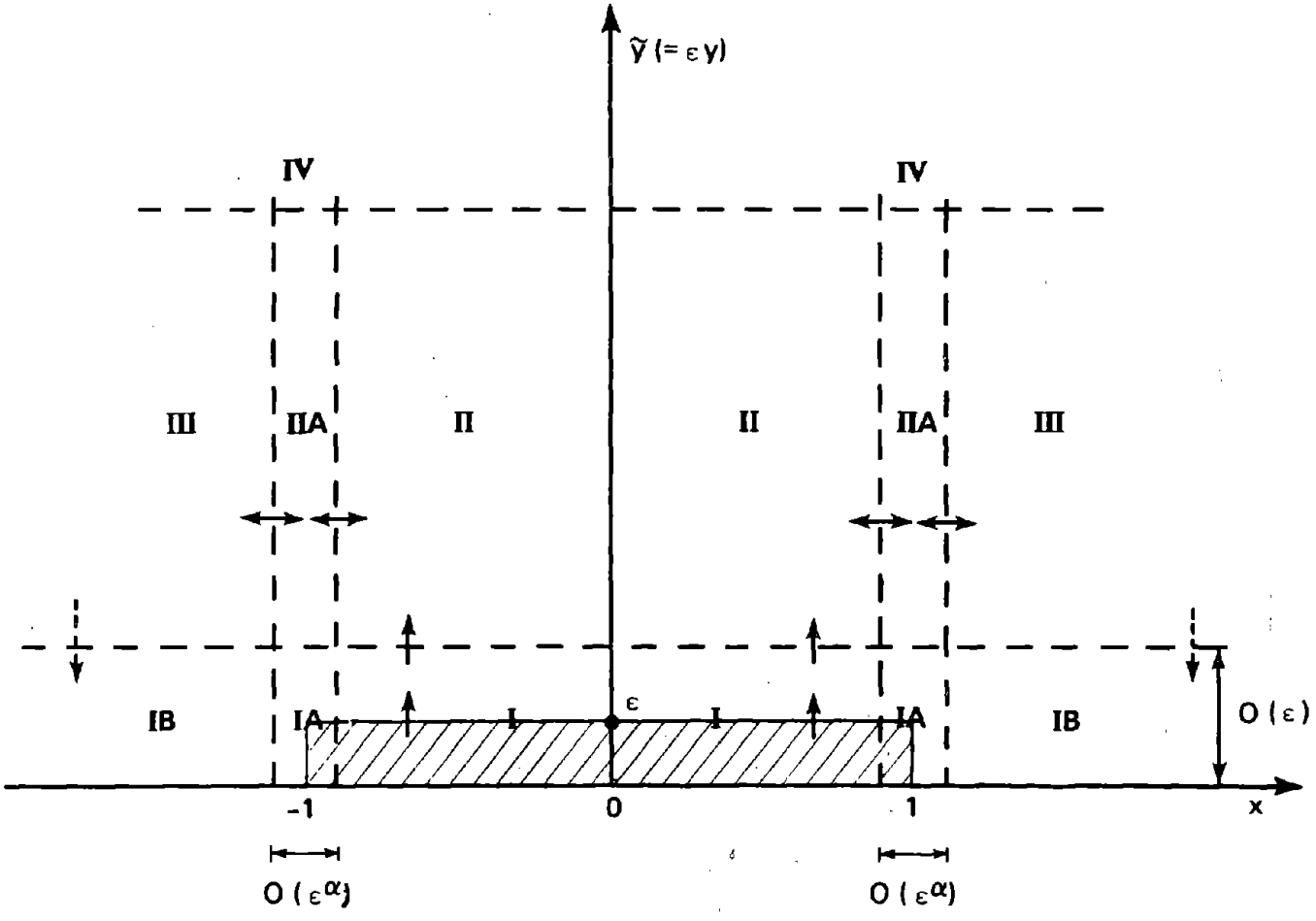


Fig. 3--Matching diagram for asymptotic solution of model equations

matched to it. Finally, expansions for the top outflow and recirculation regions (IV and III) must be successively developed and matched to the composite expansion for regions I, II, and IIA. This final matching may involve further intermediate analysis in one or both of regions IA and IB. For the moment, α (cf. Fig. 3) is left arbitrary.

The solution expansions for region I are based directly on equations (2.8). In other regions, expansions are derived from rescaled versions of those equations. In all regions, however, the expansions have the same general form--namely,

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots ;$$

$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \varepsilon^3 v_3 + \cdots ;$$

$$P = P_0 + \varepsilon P_1 + \varepsilon^2 P_2 + \varepsilon^3 P_3 + \cdots ;$$

$$\rho = \rho_0 + \varepsilon \rho_1 + \varepsilon^2 \rho_2 + \varepsilon^3 \rho_3 + \cdots ;$$

$$T = T_0 + \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + \cdots . \qquad (2.10)$$

Below, we focus on determining the leading-order terms in the expansions. Equations for those terms (which describe the basic flow structure for a large area fire) are introduced and solutions discussed.

COMBUSTION LAYER

Substituting Eq. (2.10) into Eq. (2.8) produces the following leading-order equation set for region I:

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 ;$$

$$\rho_0 \left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) = - B \frac{\partial P_1}{\partial x} + M_{11} \frac{\partial^2 u_0}{\partial x^2} + M_{12} \frac{\partial^2 u_0}{\partial y^2} ;$$

$$\frac{\partial P_1}{\partial y} + A\rho_0 = 0 ;$$

$$\rho_0 \left(u_0 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_0}{\partial y} \right) = q(x, y) - \sigma(T_0^4 - 1) + K_1 \frac{\partial^2 T_0}{\partial x^2} + K_2 \frac{\partial^2 T_0}{\partial y^2} ;$$

$$\rho_0 T_0 = P_0 = 1 . \qquad (2.11)$$

The momentum balances here are actually first-order correction equations. The leading-order equations $\partial P_0/\partial x = 0$ and $\partial P_0/\partial y = 0$ imply that P_0 is constant with a value of one ground-level atmosphere, and changes in pressure are at most $O(\varepsilon)$. Changes of such magnitude are consistent with experimental evidence on small, unenclosed fires [McCaffrey, 1979]. Further, since any far-field atmospheric temperature profile can vary only on the order of hundreds of degrees Kelvin over altitudes on the order of H (>>L), then $T_\infty(y) \sim 1 + O(\varepsilon y) + \cdots$ for $y = O(1)$ and is hence replaced in this equation set by one.

The phenomenological coefficients M_{ij} and K_j describe the extent of the turbulent forces. At present, it is only possible to estimate their magnitudes, relying on physical understanding of the balance of forces and crude calculations to approximate the diffusion coefficients $\mathcal{K}_{ij}$ and k_j. Phenomenological theories such as mixing-length theory can provide useful approximations for the turbulent forces in most regions (II, III, IV) of this component model, but they are not applicable to the turbulence generated by the fire in region I. For the present, we adopt the following qualitative approximation:

$$M_{12} = K_2 = 0 . \qquad (2.12)$$

This approximation may be removed in further research, but is not felt to be particularly restrictive. The approximation $M_{11} = K_1 = 0$ could alternatively be used in place of Eq. (2.12). However, we choose to proceed with Eq. (2.12), since horizontal diffusion due to turbulence certainly must be more than that in the vertical direction--at least in the central parts of the fire. The coefficients M_{11} and K_1 are then treated as parameters in the analysis of the combustion layer.

The resulting version of Eq. (2.11) must be solved subject to several types of boundary conditions. The solution must satisfy the following natural conditions:

$$\text{a. along } y = 0 , \qquad v_0 = 0 ,$$

$$\text{b. along } x = 0 , \qquad u_0 = 0 , \quad \frac{\partial T_0}{\partial x} = 0 . \tag{2.13}$$

That is, there can be no flow into or out of the ground, and the flow is symmetric about the $x = 0$ axis (implying that $\partial\rho_0/\partial x$, $\partial P_0/\partial x$, and $\partial P_1/\partial x$ as well as $\partial T_0/\partial x$ are equal to zero there). If the solution is to be of a type that can be smoothly matched with solutions from regions III and II, respectively, it must also satisfy the following conditions:

$$\text{c. along } x = 1 , \qquad T_0 = 1 ,$$

$$\text{d. as } y \to \infty , \qquad u_0 , \frac{\partial P_1}{\partial x} , \frac{\partial T_0}{\partial x} , \frac{\partial \rho_0}{\partial x} \to 0 . \tag{2.14}$$

Condition c simply represents the expectation that the temperature is atmospheric to leading order in the bottom few hundred meters of the far-field recirculation region--and hence at the outer edge of the combustion layer. The first part of condition d, $u_0 \to 0$ as $y \to \infty$, implies that the horizontal velocity decays to a lower order term (equal to or less than the vertical velocity) above the combustion zone. The remaining terms are required to approach zero because, as Eq. (2.21) later shows, $\partial P_1/\partial x \equiv \partial T_0/\partial x \equiv \partial \rho_0/\partial x \equiv 0$ in region II. Indeed, as shown in Sec. III, it turns out that T_0, $\rho_0 \to 1$ and $P_1 \sim$ constant - y, so that $\partial P_1/\partial x \to 0$ as $y \to \infty$. In view of Eq. (2.12), the no-slip condition, $u_0(x, 0) = 0$, is not required. Rescaling to include a viscous layer near $y = 0$ will be considered as a future model refinement.

The explicit analytic solution of Eq. (2.11) subject to Eqs. (2.12), (2.13), and (2.14) seems untenable. A computer code has therefore been developed to provide numerical solutions for this boundary value problem. Those solutions are discussed in Sec. III.

CONVECTION COLUMN

Since the characteristic height H of the urban fire convection column is on the order of D ($\sim 10^4$ m), its vertical spatial coordinate must be rescaled as

$$\tilde{y} = \varepsilon y \ . \tag{2.15}$$

To preserve continuity in rescaling Eq. (2.8), we nominally assume $u = 0(\varepsilon)$ in the column and introduce the further rescaling

$$\tilde{u} = \frac{u}{\varepsilon} \ , \tag{2.16}$$

so that $\tilde{u}$ is of order one. Subject to Eqs. (2.15) and (2.16), the nominal rescaling of Eq. (2.8) is then

$$\frac{\partial}{\partial x}(\rho\tilde{u}) + \frac{\partial}{\partial \tilde{y}}(\rho v) = 0 \ ;$$

$$\rho\left(\tilde{u}\frac{\partial \tilde{u}}{\partial x} + v\frac{\partial \tilde{u}}{\partial \tilde{y}}\right) = -\frac{B}{\varepsilon^3}\frac{\partial P}{\partial x} + \left(\frac{M_{11}}{\varepsilon}\right)\frac{\partial^2 \tilde{u}}{\partial x^2} + \varepsilon M_{12}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \ ;$$

$$\rho\left(\tilde{u}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial \tilde{y}}\right) = -\frac{B}{\varepsilon^3}\left(\frac{\partial P}{\partial \tilde{y}} + A\rho\right) + \left(\frac{M_{21}}{\varepsilon}\right)\frac{\partial^2 v}{\partial x^2} + \varepsilon M_{22}\frac{\partial^2 v}{\partial \tilde{y}^2} \ ;$$

$$\rho\left(\tilde{u}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial \tilde{y}}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(\tilde{u}\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial \tilde{y}}\right) + \left(\frac{K_1}{\varepsilon}\right)\frac{\partial^2 T}{\partial x^2} + \varepsilon K_2\frac{\partial^2 T}{\partial \tilde{y}^2} \ ;$$

$$P = \rho T \ . \tag{2.17}$$

(The term $q(x, y)$ is by definition zero in the convection column and $-\sigma[T^4 - T_\infty^4(y)]$ has been omitted above the combustion layer.) This version of Eq. (2.8) may in fact be appropriate for the study of the recirculation region (III) as well as the convection column, but it certainly must be further rescaled for the side mixing layer (region IIA), where turbulent shears and thermal gradients result in large horizontal derivatives. Such rescaling is discussed below (pp. 19-20).

The hierarchy of equation sets obtained in the asymptotic analysis of Eq. (2.17) of course depends on the magnitudes of the diffusion coefficients M_{ij} and K_j; but there are neither data nor correlations from which to estimate these magnitudes. In developing the equation for the convection column, we therefore proceed noting that the results may be subject to modification once definitive large-plume turbulence data become available.

As shown in Appendix B, conventional mixing-length theory suggests that in region II we let

$$\frac{M_{i1}}{\varepsilon} \, , \, \frac{K_1}{\varepsilon} = 0(\varepsilon) \; ; \qquad M_{i2}, \, K_2 << 1 \text{ for } i = 1, \, 2 \; . \tag{2.18}$$

Subject to this condition, the substitution of Eq. (2.10) into Eq. (2.17) with

$$u_0 = 0 \; ; \qquad \tilde{u} = \frac{u}{\varepsilon} = u_1 + \varepsilon u_2 + \varepsilon^2 u_3 + \cdots \tag{2.19}$$

[cf. Eq. (2.16)] results in the following leading-order equation set:

$$\frac{\partial}{\partial x} (\rho_0 u_1) + \frac{\partial}{\partial \tilde{y}} (\rho_0 v_0) = 0 \; ;$$

$$\rho_0 \left(u_1 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial \tilde{y}} \right) = - B \frac{\partial P_3}{\partial x} \; ;$$

$$\rho_0 \left(u_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial v_0}{\partial \tilde{y}} \right) = -B \left(\frac{\partial P_3}{\partial \tilde{y}} + A\rho_3 \right) \; ;$$

$$\rho_0 \left(u_1 \frac{\partial T_3}{\partial x} + v_0 \frac{\partial T_3}{\partial \tilde{y}} \right) = \left(\frac{\gamma - 1}{\gamma} \right) \left(u_1 \frac{\partial P_3}{\partial x} + v_0 \frac{\partial P_3}{\partial \tilde{y}} \right) - v_0 \left(\rho_1 \frac{\partial T_2}{\partial \tilde{y}} + \rho_2 \frac{\partial T_1}{\partial \tilde{y}} + \rho_3 \frac{\partial T_0}{\partial \tilde{y}} \right) \; ;$$

$$P_3 = \rho_0 T_3 + \rho_1 T_2 + \rho_2 T_1 + \rho_3 T_0 \; ; \tag{2.20}$$

where ρ_0, T_0, ρ_1, T_1, ρ_2, and T_2 [and P_0, P_1, and P_2 in Eqs. (2.21) through (2.23) below] are all functions of $\tilde{y}$ alone and satisfy

$$\frac{dP_0}{d\tilde{y}} + A\rho_0 = 0 \ ,$$

$$\rho_0 \frac{dT_0}{d\tilde{y}} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{\partial P_0}{\partial \tilde{y}} \ ,$$

$$P_0 = \rho_0 T_0 \ ,$$

$$P_0(0) = \rho_0(0) = T_0(0) = 1 \ ; \tag{2.21}$$

$$\frac{dP_1}{d\tilde{y}} + A\rho_1 = 0 \ ,$$

$$\rho_0 \frac{dT_1}{d\tilde{y}} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{dP_1}{d\tilde{y}} - \rho_1 \frac{dT_0}{d\tilde{y}} \ ,$$

$$P_1 = \rho_0 T_1 + \rho_1 T_0 \ ; \tag{2.22}$$

and

$$\frac{dP_2}{d\tilde{y}} + A\rho_2 = 0 \ ,$$

$$\rho_0 \frac{dT_2}{d\tilde{y}} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{dP_2}{d\tilde{y}} - \left(\rho_1 \frac{dT_1}{d\tilde{y}} + \rho_2 \frac{dT_0}{d\tilde{y}}\right) \ ,$$

$$P_2 = \rho_0 T_2 + \rho_1 T_1 + \rho_2 T_0 \ . \tag{2.23}$$

It can easily be shown that the solution for Eq. (2.21) is

$$T_0 = \left[1 - A\left(\frac{\gamma - 1}{\gamma}\right)\tilde{y}\right] \ ,$$

$$\rho_0 = \left[1 - A\left(\frac{\gamma - 1}{\gamma}\right)\tilde{y}\right]^{\left(\frac{1}{\gamma-1}\right)} \ ,$$

$$P_0 = \left[1 - A\left(\frac{\gamma - 1}{\gamma}\right)\tilde{y}\right]^{\left(\frac{\gamma}{\gamma-1}\right)} \ . \tag{2.24}$$

That is, as expected, the zeroth-order thermodynamic state is an adiabatic atmosphere and the column is only weakly buoyant. Subject to prescriptions of P_1, T_1, P_2, and T_2 at $y = 0$, which must come from lower order analysis of the combustion layer equations, the linear systems in Eqs. (2.22) and (2.23) are then easily solved, and the description of the basic region II flow reduces to the solution of Eq. (2.20). This equation set is to be solved subject to

$$\begin{aligned} &\text{a. along } \tilde{y} = 0 , \qquad v_0 = v_\infty(x),\ u_1 = u_{1\infty}(x), \text{ and (say)} \\ &\qquad\qquad\qquad\qquad P_3 = P_{3\infty}(x),\ T_3 = T_{3\infty}(x) , \\ &\text{b. along } x = 0 , \qquad u_1 = 0 , \end{aligned} \tag{2.25}$$

where

$$v_\infty(x) = \lim_{y\to\infty} [v_0(x, y)]_{\text{region I}} \tag{2.26}$$

and $u_{1\infty}(x)$, $P_{3\infty}(x)$, and $T_{3\infty}(x)$ are to be similarly defined (subject to lower order analysis of the combustion layer equations). The edge of the convection column is marked (to leading order) by the free streamline that begins at $(x, \tilde{y}) = (1, 0)$ and is defined from the solution of Eqs. (2.20) and (2.25).

In region IIA (i.e., in the thin layer along the $y = \eta(y)$ streamline), the differences in velocities and thermodynamic states between the main convection column (region II) and the recirculation region (III) must be smoothed out. To leading order, then, appropriately rescaled equations for the flow in this region must represent a balance between the effects of buoyancy, mixing, and convection. That balance is obtained if the horizontal coordinate is rescaled as

$$\tilde{x} = \left(\frac{x - \eta(y)}{\sqrt{\epsilon}} \right) , \tag{2.27}$$

so that the characteristic mixing-layer thickness is $O(\varepsilon^{1/2})$ (i.e., defining α in Fig. 3 as 1/2). As shown in Appendix B, the resulting leading-order equations for this layer are then

$$\frac{\partial}{\partial \tilde{x}} (\rho_0 \hat{u}_{3/2}) + \frac{\partial}{\partial \tilde{y}} (\rho_0 v_0) = 0 \ ;$$

$$u_1 = \left(\frac{d\eta}{d\tilde{y}}\right) v_0 \ ;$$

$$\rho_0 \left(\hat{u}_{3/2} \frac{\partial v_0}{\partial \tilde{x}} + v_0 \frac{\partial v_0}{\partial \tilde{y}}\right) = -B \left(\frac{dP_3}{d\tilde{y}} + A\rho_3\right) + \tilde{M}_{21} \frac{\partial^2 v_0}{\partial \tilde{x}^2} \ ;$$

$$\rho_0 \left(\hat{u}_{3/2} \frac{\partial T_3}{\partial \tilde{x}} + v_0 \frac{\partial T_3}{\partial \tilde{y}}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(v_0 \frac{dP_3}{d\tilde{y}}\right) + \tilde{K}_1 \frac{\partial^2 T_3}{\partial \tilde{x}^2} - v_0 \left(\rho_1 \frac{\partial T_2}{\partial \tilde{y}} + \rho_2 \frac{\partial T_1}{\partial \tilde{y}} + \rho_3 \frac{\partial T_0}{\partial \tilde{y}}\right) \ ;$$

$$P_3 = \rho_0 T_3 + \rho_1 T_2 + \rho_2 T_1 + \rho_3 T_0 \ ; \qquad (2.28)$$

where ρ_0, T_0, ρ_1, T_1, ρ_2, T_2, and P_3 [and P_0, P_1, and P_2] are all functions of $\tilde{y}$ alone, and hence are as in region II [cf. Eqs. (2.21) through (2.24)]. The revised expansions for $\tilde{u}$ and v [cf. Eqs. (2.10) and (2.19)] are

$$\tilde{u} = u_1 + \varepsilon^{1/2} u_{3/2} + \varepsilon u_2 + \cdots \ ,$$

$$v = v_0 + \varepsilon^{1/2} v_{1/2} + \varepsilon v_1 + \cdots \ , \qquad (2.29)$$

with

$$\hat{u}_{3/2} = u_{3/2} - \left(\frac{d\eta}{dy}\right) v_{1/2} \ , \qquad (2.30)$$

and

$$\tilde{M}_{i1} = \left(\frac{M_{i1}}{\varepsilon^2}\right) = O(1) \qquad \text{for } i = 1, 2 ,$$

$$\tilde{K}_1 = \left(\frac{K_1}{\varepsilon^2}\right) = O(1) \tag{2.31}$$

[cf. Eq. (2.18)]. In order that a solution of Eq. (2.28) match (as $\tilde{x} \to -\infty$) with the region II solution of Eq. (2.20) [as $x \to \eta(\tilde{y})$], it must satisfy the boundary conditions

$$\lim_{\tilde{x}\to-\infty} v_0(\tilde{x}, \tilde{y}) = \lim_{x\to\eta(\tilde{y})} v_0^{(II)}(x, \tilde{y}) ,$$

$$\lim_{\tilde{x}\to-\infty} \left[\frac{\partial u_{3/2}}{\partial \tilde{x}} (\tilde{x}, \tilde{y})\right] = \lim_{x\to\eta(\tilde{y})} \left[\frac{\partial u_1^{(II)}}{\partial x} (x, \tilde{y})\right] ,$$

$$\lim_{\tilde{x}\to-\infty} T_3(\tilde{x}, \tilde{y}) = \lim_{x\to\eta(\tilde{y})} T_3^{(II)}(x, \tilde{y}) , \tag{2.32}$$

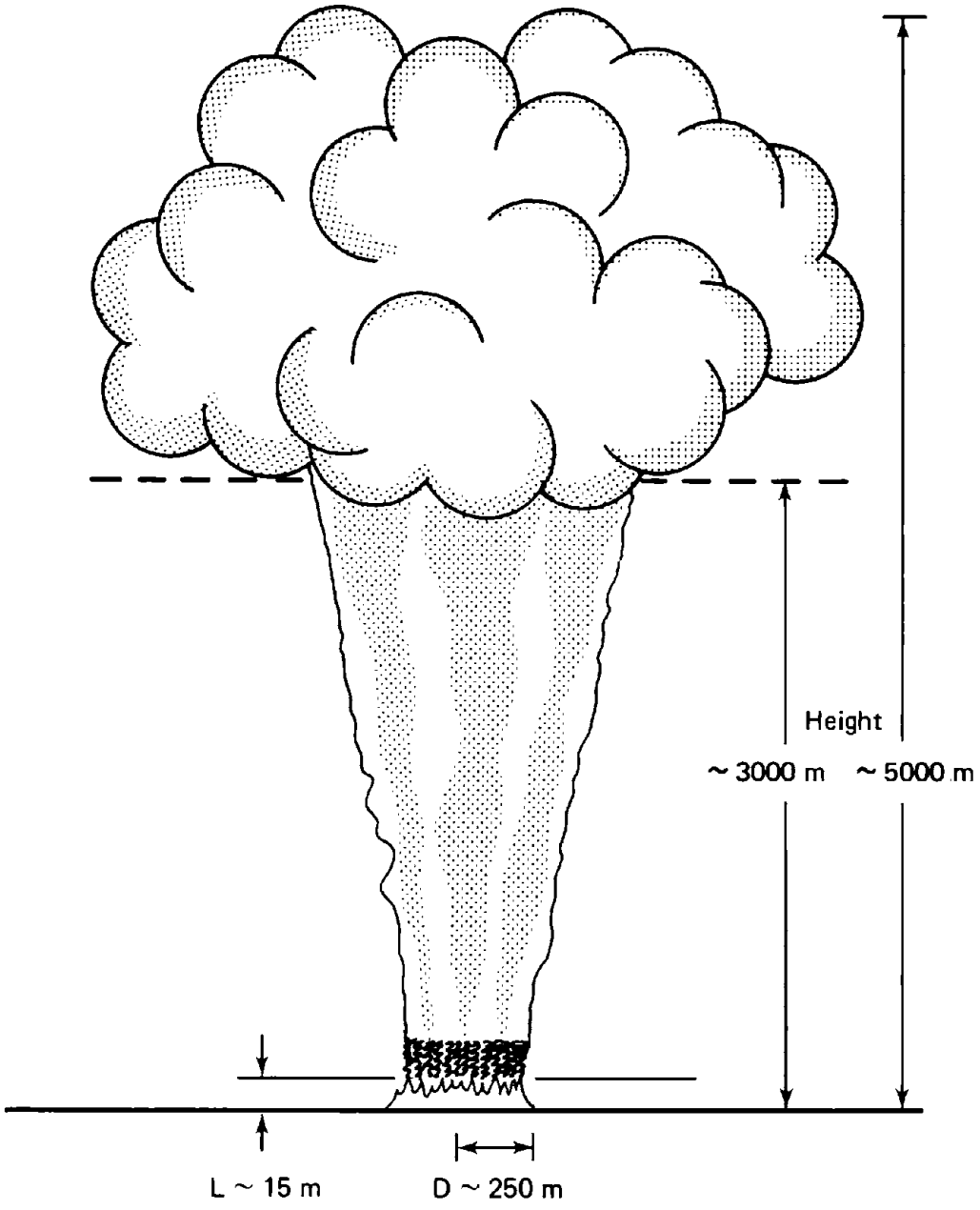
where the superscript (II) refers to the region II solution. Similar boundary conditions are required (as $\tilde{x} \to \infty$) for the solution of Eq. (2.28) to be matched to the solution of the equations that are ultimately introduced for the recirculation region. The solution of Eq. (2.28) subject to those boundary conditions must presumably be obtained by numerical methods. This task is left for future research.

SPECIAL CASE OF FLAMBEAU FIRES

The analysis presented so far is primarily applicable to a large area fire (i.e., $\varepsilon \to 0$) with a convection-column aspect ratio $D/H \sim O(1)$. Here we apply the theory to the much smaller multiple-fuel-bed Flambeau fire [Countryman, 1969] illustrated in Fig. 4. Our analysis is based on the matching diagram in Fig. 3, but omits the side mixing layer, region IIA.

As shown in Fig. 4, $L \sim 15$ m, $D \sim 250$ m, and

$$\varepsilon = 6 \times 10^{-2} .$$



Source: Countryman [1969].

Fig. 4--Sketch of a typical multiple-fuel-bed Flambeau fire

From Eq. (2.6), the indicated scale velocity is

$$U = 11.2 \text{ m/sec} ;$$

and, from Eq. (2.4), $\delta_1 \sim O(\varepsilon^2)$ and $\delta_2 \sim O(\varepsilon)$. By applying these scalings, the basic equation set is found to be the following slight revision of Eq. (2.8):

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 ;$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\left(\frac{B}{\varepsilon^2}\right)\frac{\partial P}{\partial x} + M_{11}\frac{\partial^2 u}{\partial x^2} + M_{12}\frac{\partial^2 u}{\partial y^2} ;$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\left(\frac{B}{\varepsilon^4}\right)\left(\frac{\partial P}{\partial y} + \varepsilon^2 A\rho\right) + M_{21}\frac{\partial^2 v}{\partial x^2} + M_{22}\frac{\partial^2 v}{\partial y^2} ;$$

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + q(x, y) - \sigma[T^4 - T_\infty^4(y)]$$

$$+ K_1\frac{\partial^2 T}{\partial x^2} + K_2\frac{\partial^2 T}{\partial y^2} ;$$

$$P = \rho T ; \qquad (2.33)$$

where $A = \delta_1/\varepsilon\delta_2 \sim 0.52$, $B = \varepsilon^2/\delta_1 \sim 2.3$, and $\sigma \sim 0.023$ (for $k^{*-1} \sim$ 15 m). The coefficients M_{ij} and K_j are taken to be of order one (since our analysis again shows that turbulent mixing is required for thick plume formation).

Substitution of Eq. (2.10) into Eq. (2.33) shows that the leading-order combustion-layer equation set for the Flambeau fire is that for large-scale urban fires, namely Eq. (2.11), except for changes in A, B, and σ (and possibly the diffusion coefficients). For convenience, we also make the qualitative simplification of Eq. (2.12) and seek a solution for the resulting equation set that satisfies the boundary conditions Eqs. (2.13) and (2.14).

We also, however, prescribe one further boundary condition for the Flambeau fire model. Prescription of u or $\partial u/\partial x$ is required along x = 1 (the inlet edge of the combustion region) if the solution for Eqs. (2.11), (2.12), (2.13), and (2.14) is to be unique. In general (cf. Sec. III), the inlet velocities should depend on the heat release and buoyancy generated in the combustion zone. That dependence holds even if the boundary prescription is applied as $x \to \infty$ [Smith, Morton, and Leslie, 1975].

For the Flambeau fire of Fig. 4, however, it appears that a reasonably accurate definition of u along x = 1 can be made from experimental data collected at several points on the fire periphery [Countryman, 1969]. Though these data are subject to scatter and may reflect some experimental error, horizontal velocity components at all collection points seem fairly constant (at about 10 m/sec) over the average flame height (~15 m). They then begin to decrease with increasing height above the flame region. The data suggesting such a decrease are for heights up to about 20 m. An exponential decrease above the fire zone is assumed, and the boundary condition is constructed as

$$\hat{c}. \quad \text{along } x = 1 , \qquad u = \begin{cases} u_I , & \text{for } 0 \le y \le 1 \\ u_I \, e^{-(y-1)} , & \text{for } y > 1 \end{cases} ,$$

$$u_I \sim 0.895 \ (\equiv 10 \text{ m/sec}) . \qquad (2.34)$$

Numerical simulation shows that the combustion layer dynamics are fairly insensitive to quantitative changes in the exponential decay factor.

Since $D/H \sim O(\varepsilon)$, the appropriate rescaling of Eq. (2.33) for the Flambeau convection-column region involves

$$\tilde{y} = \varepsilon^2 y ; \qquad \tilde{u} = \frac{u}{\varepsilon^2} = u_2 + \varepsilon u_3 + \cdots . \qquad (2.35)$$

The resulting equations describing the convection column are

$$\frac{\partial}{\partial x}(\rho\tilde{u}) + \frac{\partial}{\partial \tilde{y}}(\rho v) = 0 \ ;$$

$$\rho\left(\tilde{u}\frac{\partial \tilde{u}}{\partial x} + v\frac{\partial \tilde{u}}{\partial \tilde{y}}\right) = -\left(\frac{B}{\varepsilon^6}\right)\frac{\partial P}{\partial x} + \frac{M_{11}}{\varepsilon^2}\frac{\partial^2 \tilde{u}}{\partial x^2} + \varepsilon^2 M_{12}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \ ;$$

$$\rho\left(\tilde{u}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial \tilde{y}}\right) = -\left(\frac{B}{\varepsilon^4}\right)\left(\frac{\partial P}{\partial \tilde{y}} + A\rho\right) + \frac{M_{21}}{\varepsilon^2}\frac{\partial^2 v}{\partial x^2} + \varepsilon^2 M_{22}\frac{\partial^2 v}{\partial \tilde{y}^2} \ ;$$

$$\rho\left(\tilde{u}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial \tilde{y}}\right) = \left(\frac{\gamma - 1}{\gamma}\right)\left(\tilde{u}\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial \tilde{y}}\right) + \frac{K_1}{\varepsilon^2}\frac{\partial^2 T}{\partial x^2} + \varepsilon^2 K_2\frac{\partial^2 T}{\partial \tilde{y}^2} \ ;$$

$$P = \rho T \ . \qquad (2.36)$$

Since the convection-column aspect ratio in this case is small [i.e., $D/H = O(\varepsilon)$], the column flow is expected to be that of a "standard" plume [Murgai and Emmons, 1960]. Indeed, Eq. (2.36) can be reduced to the usual set of plume equations in the following way. With $D \ll H$, the spread of energy and momentum horizontally is expected to be much greater than that in the vertical direction, and is assumed to balance the convective transfer of these quantities. The vertical diffusion terms [Eq. (2.36)] are therefore neglected and the following order estimates are made:

$$\frac{M_{11}}{\varepsilon^2} = \tilde{M}_{11} = O(1) \ , \qquad \frac{M_{21}}{\varepsilon^2} = \tilde{M}_{21} = O(1) \ ,$$

$$\frac{K_1}{\varepsilon^2} = \tilde{K}_1 = O(1) \ . \qquad (2.37)$$

It is then inferred from the horizontal momentum equation [Eq. (2.36)] that P is a function of $\tilde{y}$ alone through at least five orders in ε. Hence, from the vertical momentum balance, this function,

$$\tilde{P}(\tilde{y}) = \sum_{n=0}^{5} \varepsilon^n P_n \ ,$$

satisfies

$$\frac{d\tilde{P}}{d\tilde{y}} = -A\rho_\infty(\tilde{y}) \ , \tag{2.38}$$

where $\rho_\infty(\tilde{y})$ is the limiting density profile through five orders in ε as $|x| \to \infty$ (and $\tilde{u}$, $v \to 0$). Defining

$$\Gamma = \left(\frac{AB}{\varepsilon^4}\right)\rho \ , \qquad \Gamma_\infty = \left(\frac{AB}{\varepsilon^4}\right)\rho_\infty \ , \tag{2.39}$$

and using Eqs. (2.37) and (2.38), then the following equation set is found to be valid to leading order in an $\varepsilon \to 0$ expansion:

$$\frac{\partial}{\partial\tilde{x}}(\rho\tilde{u}) + \frac{\partial}{\partial\tilde{y}}(\rho v) = 0 \ ;$$

$$\tilde{u}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial\tilde{y}} = \frac{1}{\rho}(\Gamma_\infty - \Gamma) + \frac{1}{\rho}\left(\tilde{M}_{21}\frac{\partial^2 v}{\partial x^2}\right) \ ;$$

$$\tilde{u}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial\tilde{y}} + A\left(\frac{\gamma - 1}{\gamma}\right)v = \frac{1}{\rho}\left(\tilde{K}_1\frac{\partial^2 T}{\partial x^2}\right) \ . \tag{2.40}$$

These are exactly the cartesian coordinate equivalents of the "standard" buoyant plume equations.

Similarity solutions of Eqs. (2.40) [Morton, Taylor, and Turner, 1956; Murgai and Emmons, 1960] require definition of the fire size, atmosphere, initial vertical velocity profile, initial buoyancy ($\Gamma_\infty - \Gamma$), and either an entrainment law or mixing-length hypothesis. Solution of the leading-order combustion-layer equation set [Eqs. (2.11) subject to Eqs. (2.12) through (2.14)] defines the vertical velocity profile. Prescription of the initial buoyancy requires lower order analysis of the combustion region. Computed results for the 250 m Flambeau combustion layer are presented in Sec. III.

COMBUSTION PRODUCTS

Once the heat release rate $Q \cdot q(x, y)$ is prescribed, the release rates for all other combustion end-products and the corresponding depletion rate for oxygen can be computed (at least approximately), assuming stoichiometric (or excess air) burning conditions. Simple uncoupled equations of mass conservation for individual reactant species can then be introduced and solved. For oxygen, carbon monoxide, carbon dioxide, and "smoke" (i.e., suspended solids)--four important reactant species in the basic large urban fire environment--these equations are

$$\rho\left(u\frac{\partial\phi_m}{\phi x} + v\frac{\partial\phi_m}{\phi y}\right) = \mathcal{S}_m \cdot \mathcal{R}_m(x, y) + D_{1m}\frac{\partial^2\phi_m}{\partial x^2} + D_{2m}\frac{\partial^2\phi_m}{\partial y^2}\,, \qquad (2.41)$$

where m refers individually to oxygen, carbon monoxide, carbon dioxide, and smoke, and ϕ_m is the mass fraction of m in the flow. The various $\mathcal{S}_m \cdot \mathcal{R}_m(x, y)$ are source/sink rates due to the combustion processes, and the D_{1m} are turbulent diffusion coefficients. Consistent with the preceding analysis of Eq. (2.11), the qualitative approximation

$$D_{2m} = 0 \qquad (2.42)$$

is made [cf. Eq. (2.12)]. The various D_{1m} are assumed to be order-one parameters.

To leading order (as $\varepsilon \to 0$), the solution of Eq. (2.41) is to be found subject to

$$u = u_0\,, \quad v = v_0\,, \quad \rho = \rho_0\,, \qquad (2.43)$$

u_0, v_0, and ρ_0 coming from the solution of Eq. (2.11). The natural boundary conditions (using the same letter designations as on p. 15) for Eq. (2.41) are

b. along $x = 0$, $\dfrac{\partial \phi_m}{\partial x} = 0$, for all m,

c. along $x = 1$, $\phi_m = 0.209$, for $m = O_2$,

$\phi_m = 0$, for $m = CO, CO_2$, smoke (2.44)

(i.e., symmetry at the center line and atmospheric values at the inlet edge). In view of Eq. (2.43), Eq. (2.41) is linear, and the boundary value problem is easily solved (numerically) once the ϕ_m are known along $y = 0$. But this information is also easily obtained. For $y = 0$, $v = 0$; thus, subject to Eq. (2.42), the leading-order version of Eq. (2.41) reduces to

$$(\rho_0 u_0) \frac{\partial \phi_m}{\partial x} = \mathcal{S}_m \cdot \mathcal{R}_m(x, 0) + D_{1m} \frac{\partial^2 \phi_m}{\partial x^2}, \tag{2.45}$$

which can then be solved subject to Eq. (2.44) as a standard ordinary-differential-equation, boundary-value problem. After the basic variables ρ_0, u_0, and v_0 are computed, predictions of oxygen depletion and carbon monoxide, carbon dioxide, and smoke contamination due to large fires can thus be made by solving Eq. (2.45) and then the leading-order versions of Eqs. (2.41) and (2.44).

CYLINDRICAL SYMMETRY

The procedure used in solving the two-dimensional problem can easily be adapted to the case of cylindrical symmetry. Subject again to Eq. (2.12), the leading-order equation set has the following form for a cylindrically symmetric fire:

$$\frac{\partial}{\partial r}(\rho_0 u_0 r) + \frac{\partial}{\partial y}(\rho_0 v_0 r) = 0 ;$$

$$\rho_0 \left(u_0 \frac{\partial u_0}{\partial r} + v_0 \frac{\partial u_0}{\partial y} \right) = -B \frac{\partial P_1}{\partial r} + M_{11} \left(\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r} \frac{\partial u_0}{\partial r} \right) ;$$

$$\frac{\partial P_1}{\partial y} + A\rho_0 = 0 \ ;$$

$$\rho_0 \left(u_0 \frac{\partial T_0}{\partial r} + v_0 \frac{\partial T_0}{\partial y} \right) = q(x, y) - \sigma(T_0^4 - 1) + K_1 \left(\frac{\partial^2 T_0}{\partial r^2} + \frac{1}{r} \frac{\partial T_0}{\partial r} \right) \ ;$$

$$\rho_0 T_0 = 1 \ . \qquad (2.46)$$

Here, y is the vertical coordinate and r is the radial distance from the center of the fire. The boundary conditions in Eqs. (2.13) and (2.14) remain unchanged in this formulation, except that x is replaced by r.

III. COMBUSTION-LAYER BOUNDARY-VALUE PROBLEM

ANALYSIS

The use of asymptotic expansions for each region enables reduction of the conservation equations [Eq. (2.1)] to more concise statements of the balance of principal forces. For the combustion layer, the leading-order equations reduce to the following two first-order linear equations, two second-order nonlinear equations, and one equation of state:

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 \ ;$$

$$\rho_0 \left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) = - B \frac{\partial P_1}{\partial x} + M_{11} \frac{\partial^2 u_0}{\partial x^2} \ ;$$

$$\frac{\partial P_1}{\partial y} + A\rho_0 = 0 \ ;$$

$$\rho_0 \left(u_0 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_0}{\partial y} \right) = q(x, y) - \sigma(T_0^4 - 1) + K_1 \frac{\partial^2 T_0}{\partial x^2} \ ;$$

$$\rho_0 T_0 = 1 \ . \qquad (3.1)$$

Including the diffusion terms allows specification of a condition for u at both the symmetry line and the entrance to the combustion layer. An inviscid formulation [Larson and Small, 1980] results in a nontrivial (physically meaningful) solution only when x is rescaled near the centerline as $\bar{x} = x/\varepsilon^{3/2}$. This limitation implies an accelerating flow in the combustion layer, which slows and turns in the neighborhood of x = 0 and results in a high-velocity, thin (with respect to D) plume. Solutions of this type have been reported by, for example, Nielsen and Tao [1965].

Observations of large, free-burning fires [Countryman, 1969] indicate that the convection column thickness is of the same order as the horizontal fuel-bed dimension. Accordingly, we seek solutions

of Eq. (3.1) subject to the boundary conditions of Eqs. (2.13) and (2.14), and one further boundary condition along x = 1. The additional boundary condition is required because of the parabolic nature of the horizontal momentum equation. All boundary conditions are illustrated in Fig. 5.

It is clear from Eq. (3.1) that the effects of radiation and diffusion drive T_0 and ρ_0 to one as $y \to \infty$, and that $u_0 \to 0$ in this limit if $\partial P_1/\partial x$ also does. All conditions in Eq. (2.14) are thus

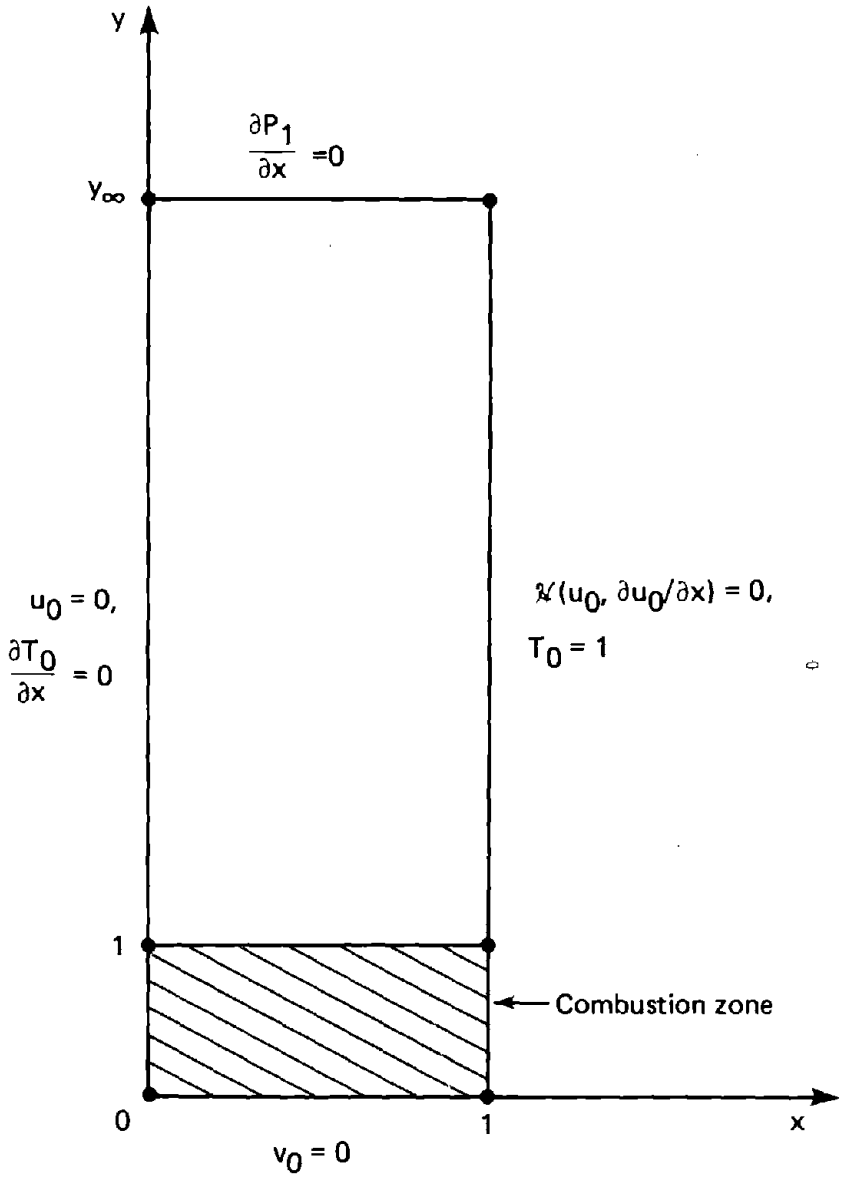


Fig. 5--Boundary conditions for leading-order combustion-layer equations

satisfied if either $u_0 = 0$ or $\partial P_1/\partial x = 0$ is required as $y \to \infty$. The latter is chosen, since one boundary condition for P_1 is needed to complement the vertical momentum equation, which is of first order in P_1. In practice, it was found sufficient to carry the calculation to $y \sim 8$ (eight flame heights) to satisfy the conditions at infinity.

The specific nature of the additional velocity boundary condition that must be posed along $x = 1$ is as yet unknown, and may be the subject of further research. The correct posing of this condition is prerequisite to answering two major practical questions: How fast are the maximum winds generated by a given fire intensity (i.e., heat release rate)? And, what is the horizontal extent of these winds beyond the fire and the associated collateral damage?

In modeling the Flambeau fire (see Sec. II, pp. 21-26), the horizontal velocity along the periphery of the burning area is defined by experimental data. For large urban fires, however, there is not yet any way to predict this velocity profile.

We suspect that the proper condition has the form

$$\mathcal{N}\left(u_0 \, , \, \frac{\partial u_0}{\partial x}\right) = 0 \, , \qquad \text{along } x = 1 \, , \tag{3.2}$$

where the functional form of $\mathcal{N}$ is to be prescribed, possibly by general integral relations relating the inlet velocity to the buoyancy, pressure, and heat release rate. This question will be addressed by future research.

Alternatively, an empirical relation similar in concept to the plume entrainment laws might be developed to relate v_0 or $\partial v_0/\partial y$ to a characteristic velocity of the local flow, e.g., $u_0(x, y)$. Such an inlet condition would have the form

$$-\frac{\partial}{\partial x}\,(\rho_0 u_0) = \frac{\partial v_0}{\partial y} = \mathcal{F}(u_0 0) \, , \qquad \text{along } x = 1 \, . \tag{3.3}$$

Recent studies [Smith, Morton, and Leslie, 1975; Cox and Chitty, 1980] have noted that the entrainment laws [$u(1, y) \propto v(0, y)$] are not valid in the neighborhood of the combustion zone. Approximate relations for the mass flow into the burning zone have been suggested by Cox and Chitty [1980]. Future experimental work may allow definition of an empirical relation resembling Eq. (3.3).

At present, there is little justification for using any specific form for either $\mathcal{N}$ or $\mathcal{F}$. Intuition suggests that $v_0(1, y)$ increases monotonically with increasing y at least to the top of the burning zone and remains constant to leading order for several flame heights above it. Accordingly, along $x = 1$, Eq. (3.3) is used to prescribe the combustion-zone inlet velocity with

$$v_0(1, y) = \hat{\alpha} \tanh (\hat{\beta} y/\hat{\alpha}) ,$$

$$\hat{\alpha} = \lim_{y \to \infty} v_0(1, y) , \qquad \hat{\beta} = \frac{\partial v_0(1, 0)}{\partial y} . \tag{3.4}$$

NUMERICAL SOLUTION

The boundary value problem posed by Eqs. (3.1), (2.13), and (2.14), together with Eqs. (3.2) or (3.3), is unusual, and its solution is interesting. With the horizontal momentum and energy equations being parabolic for u_0 and T_0, and the vertical momentum and continuity equations being first order for P_1 and v_0, respectively, a "standard" boundary value problem for Eq. (3.1) would have u_0, T_0, and P_1 as well as v_0 prescribed as "initial data" on $y = 0$, u_0 and $\partial T_0/\partial x$ along $x = 0$, and $\mathcal{N}(u_0, \partial u_0/\partial x)$ and T_0 on $x = 1$, but no restrictions at all as $y \to \infty$. In contrast, both initial and "final" ($y = y_\infty$) data are prescribed (cf. Fig. 5), the difference between the two formulations being analogous to that between initial value problems and two-point boundary value problems for ordinary differential equations.

A standard method for solving two-point boundary value problems--"shooting" [Keller, 1968]--has thus been modified to provide numerical solutions to the problem posed above. The resulting scheme,

based on using solutions of the "standard" ("initial data") problem, is outlined in Fig. 6 and detailed in Appendix C.

The solution algorithm has been implemented on the Perkin-Elmer 3220 using both a prescribed inlet velocity and experimental data. In the first case, the numerical solution to the boundary value problem [Eqs. (3.1), (2.13), and (2.14)] is subject to the inlet-velocity boundary condition

$$\frac{\partial}{\partial x}(\rho_0 u_0) = \hat{\phi}(y) + \beta_0 u_0 , \qquad \text{along } x = 1 , \tag{3.5}$$

with $\hat{\phi}(y)$ and β_0 prescribed as inputs. This case includes the general condition of Eq. (3.4), with

$$\hat{\phi}(y) = -\frac{\partial v_0}{\partial y} , \qquad \beta_0 = 0 , \tag{3.6}$$

and a special (sample) form of Eq. (3.3), where

$$\mathfrak{F}(u_0) = \beta_0 u_0 , \qquad \hat{\phi}(y) = 0 . \tag{3.7}$$

In the second case, the numerical solution is subject to

$$u_0 = \hat{g}(y) , \qquad \text{given, along } x = 1 , \tag{3.8}$$

where $\hat{g}(y)$ is defined by the Flambeau fire experimental data [cf. Eq. (2.34)].

Stable, convergent computations have been carried out in both cases, complete solutions being generated in 4 to 12 min. However, convergence requires reasonable initial guesses for $P_1(x, 0)$, and also $T_0(x, 0)$ and $u_0(x, 0)$--normal restrictions when nonlinear equations are solved by iterative methods. These codes are documented and further discussed in Appendix C.

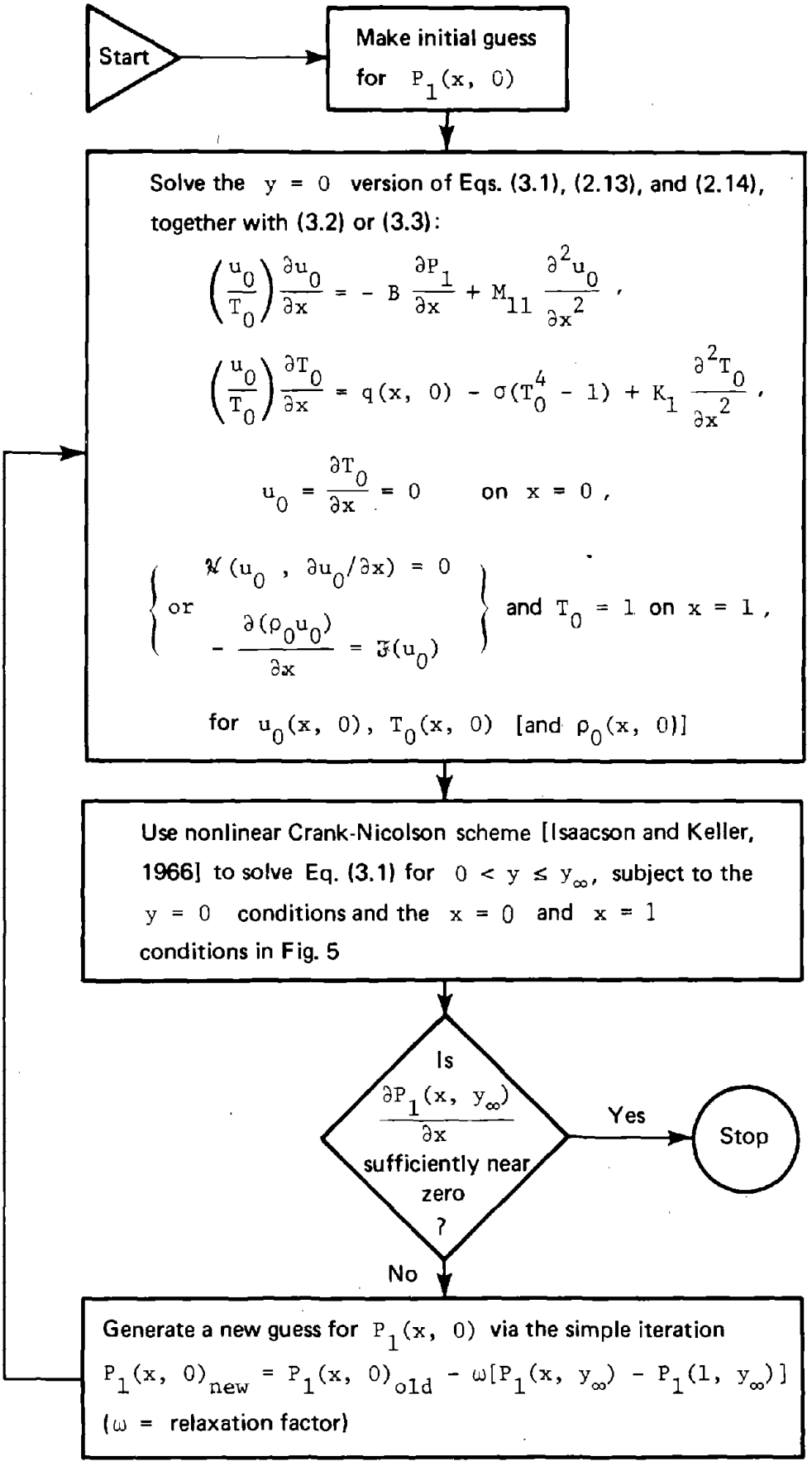


Fig. 6--Macro flowchart of solution algorithm for basic combustion-layer problem

RESULTS

A beginning series of parametric studies for the combustion-layer flow field, though incomplete, explains the gas dynamics in and near the burning region. These calculations explore some variations in the winds at the boundary, the turbulent viscosity, the heat diffusivity (due to turbulent mixing), the radiation losses, and the burning rate. Table 1 presents a partial list of calculations.

Table 1

PARAMETER COMBINATIONS CONSIDERED

Case	$\hat{\alpha}$	$\hat{\beta}$	σ	M_{11}	K_1	Remarks
A	1.0	1.5	0.023	0.5	0.5	Baseline case: $q(x, y) \equiv 1$
B	1.5	1.5	0.023	0.5	0.5	Higher vertical velocity input
C	0.5	1.5	0.023	0.5	0.5	Lower vertical velocity input
D	1.0	2.0	0.023	0.5	0.5	Increased gradient of vertical velocity input
E	1.0	1.0	0.023	0.5	0.5	Decreased gradient of vertical velocity input
F	1.0	1.5	0.05	0.5	0.5	Radiation increased
G	1.0	1.5	0.01	0.5	0.5	Radiation decreased
H	1.0	1.5	0.023	1.5	1.5	Diffusion increased
I	1.0	1.5	0.023	0.25	0.25	Diffusion decreased
J	1.0	1.5	0.023	0.10	0.10	Diffusion decreased
K	1.0	1.5	0.023	0.2	0.8	Variable diffusion coefficients
L	1.0	1.5	0.023	0.8	0.2	Variable diffusion coefficients
M	1.0	1.5	0.023	0.5	0.5	Double heating rate: $q(x, y) \equiv 2$
N	1.0	1.5	0.023	0.5	0.5	Halved heating rate: $q(x, y) \equiv 0.5$
O	1.0	1.5	0.023	0.5	0.5	Periodic fires (2): $q(x, y) = 1 - \cos 4\pi x$
P	1.0	1.5	0.023	0.5	0.5	Periodic fires (10): $q(x, y) = 1 - \cos 20\pi x$
Q	1.0	1.5	0.023	0.5	0.5	Ring ("inner dead zone") fire: $q(x, y) = 1.33$; $1/3 \leq x \leq 1$

The variations represented by runs A, B, C, D, and E explore the influence of the boundary condition for vertical velocities at the outer edge of the fire and convection column. The horizontal inflow velocities at $x = 1$ are computed as part of the solution, and not *a priori* defined. The assumed form of the vertical velocity on the outer boundary is given by Eq. (3.4), in which the asymptotic value of the velocity

with height $y \to \infty$ is represented by $\hat{\alpha}$ and the initial slope of the velocity profile with altitude $y = 0$ is represented by $\hat{\beta}$. Figure 7 illustrates the boundary vertical velocity versus altitude for cases A through E. Case C has the lowest asymptotic velocity; case A, the standard intermediate value; and case B, the highest value. Similarly, case D has the highest initial slope, case A the intermediate slope, and case E the lowest or slowest initial slope.

Figure 8 compares the velocity fields for the three cases A, B, and C, but is rather difficult to interpret because the plots do not have the same scale for the velocities. The maximum velocities are approximately in the ratio 2:3:4 = C:A:B. One can deduce for case C (lowest boundary velocity) that the motion in the plume above the fire region is fairly vertical and has less horizontal inflow than either case A or B, where the boundary vertical velocities are higher and the horizontal velocities at $x = 1$ are appreciable well above the fire. It appears that the larger the vertical velocities at the boundary, the greater the mass flux into the plume well above the fire. The horizontal velocities coming in at the base of the fire are all approximately equal ($u \sim 2$), after the different scales in the plots have been normalized. The boundary condition is chosen somewhat arbitrarily, but it is evident that a qualitatively similar flow pattern develops for a fairly wide range of boundary velocities.

Figure 9 compares the temperature and pressure contours for cases A, B, and C. The temperature gradients increase as velocity decreases ($\hat{\alpha}$), but the pressure gradients appear to have equalized across the early plume somewhat more rapidly for the lowest-boundary-velocity case C.

Figure 10 compares temperature and velocity along the central axis of the fire for the same three cases. The center-line velocity continues to increase above the actual burning region in case B, where the boundary velocity is highest. The velocity along the center line levels off and increases only slightly above the burning region for the baseline calculation (case A). The velocity actually decreases above the top of the burning region in case C, where the boundary velocity is the least. The increased temperature peak in case C reflects

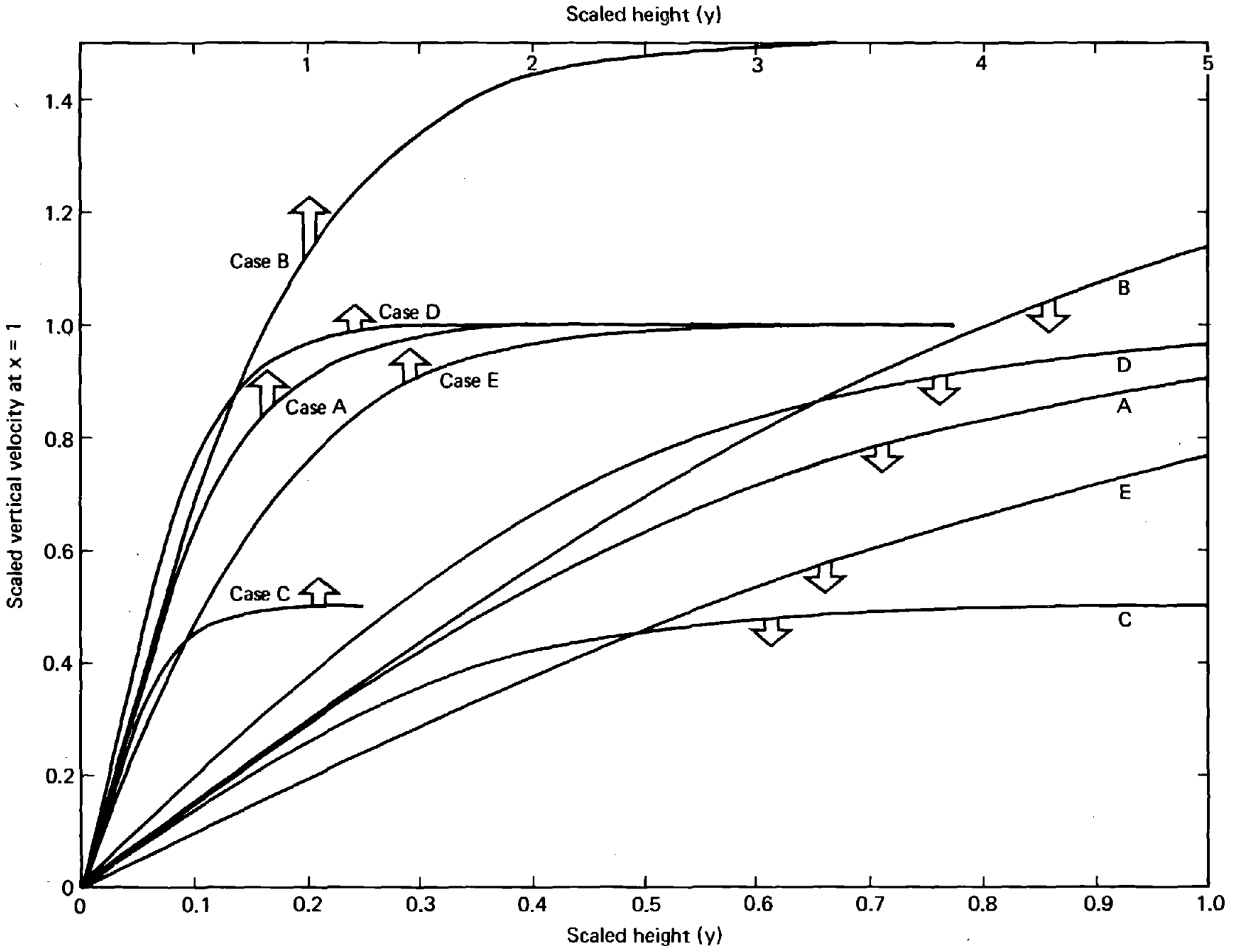


Fig. 7--Vertical velocity profiles used along inlet boundary

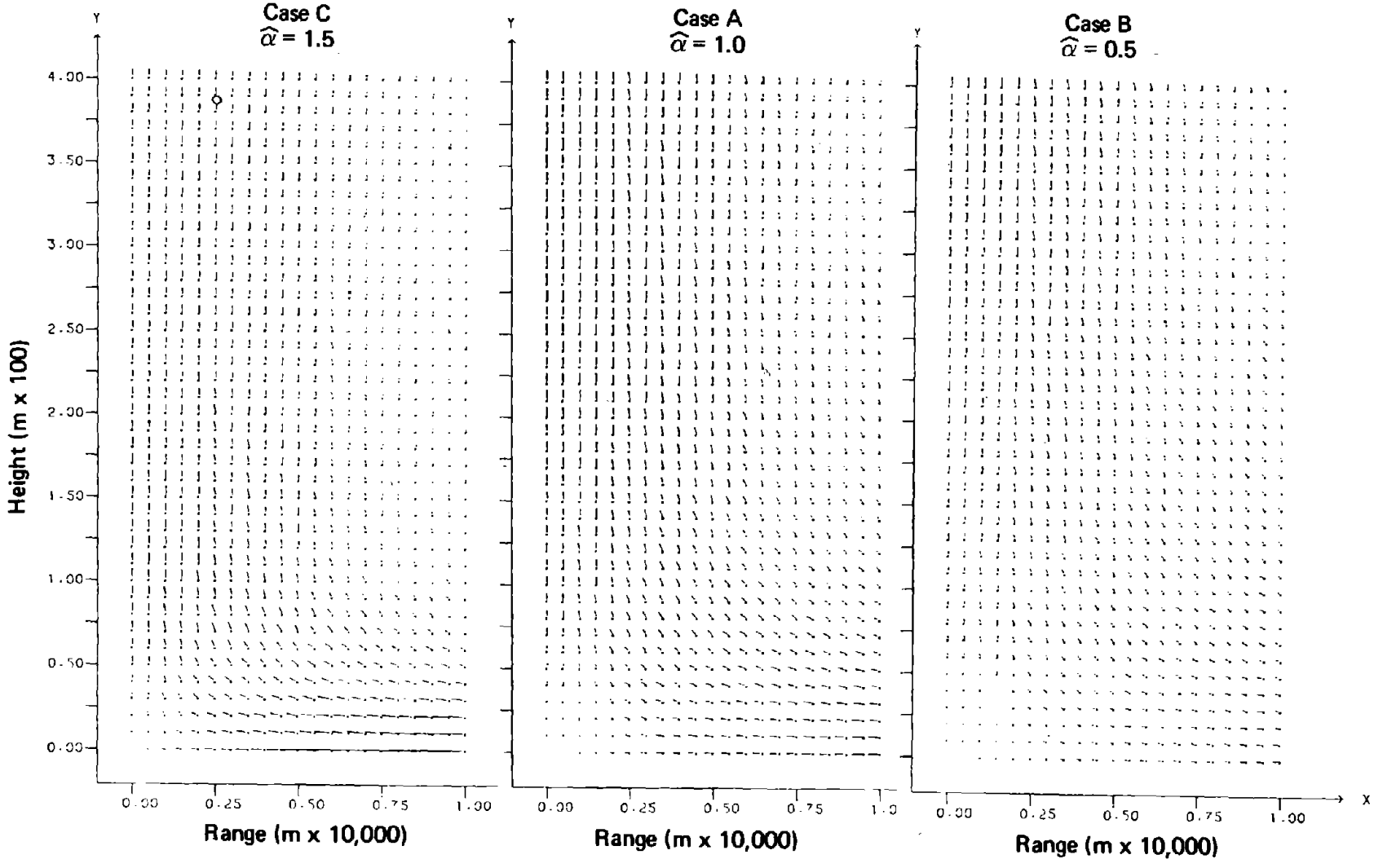


Fig. 8--Velocity fields generated for different inlet velocity magnitudes

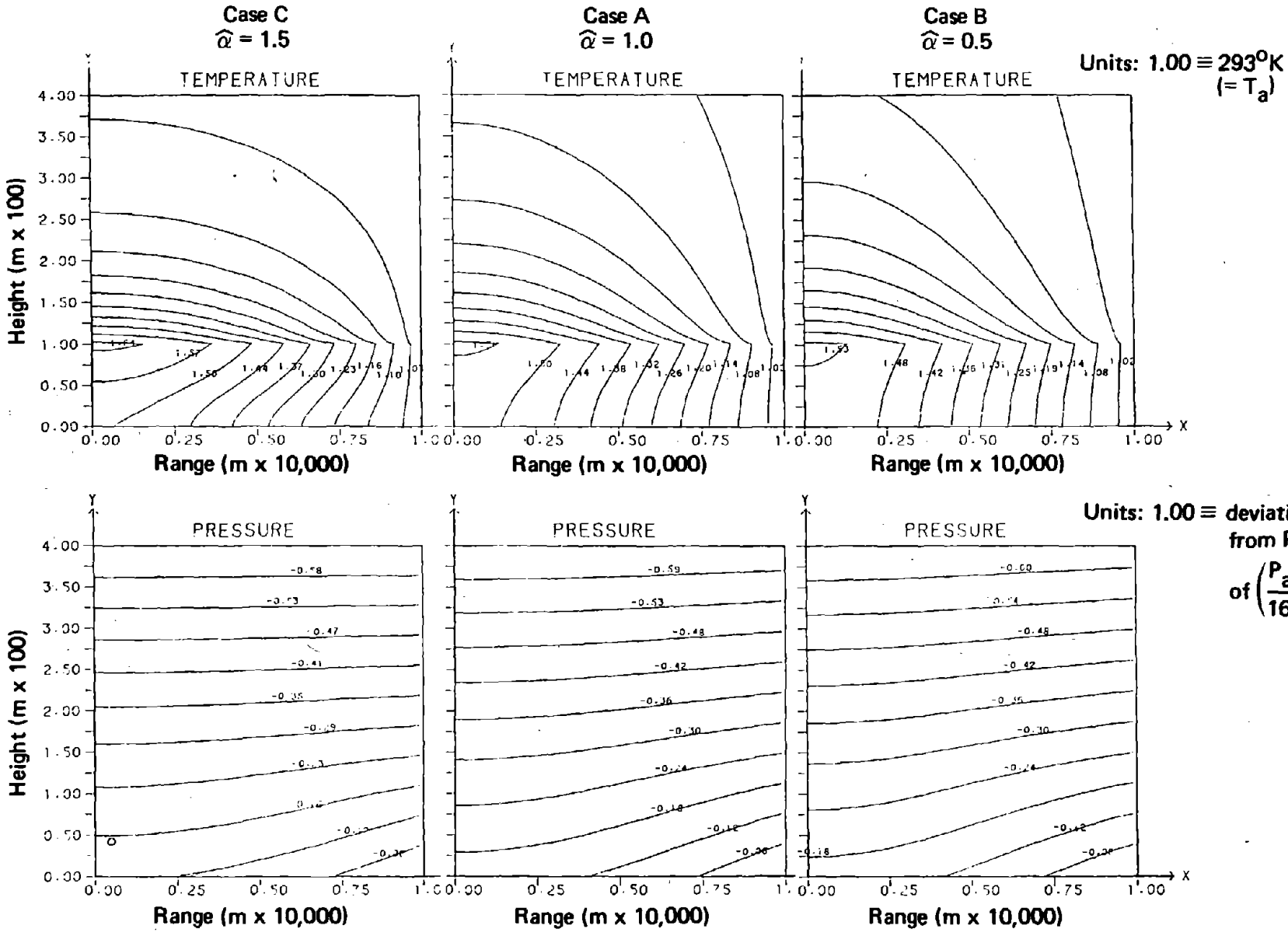


Fig. 9--Temperature and pressure contours generated for different inlet velocity magnitudes

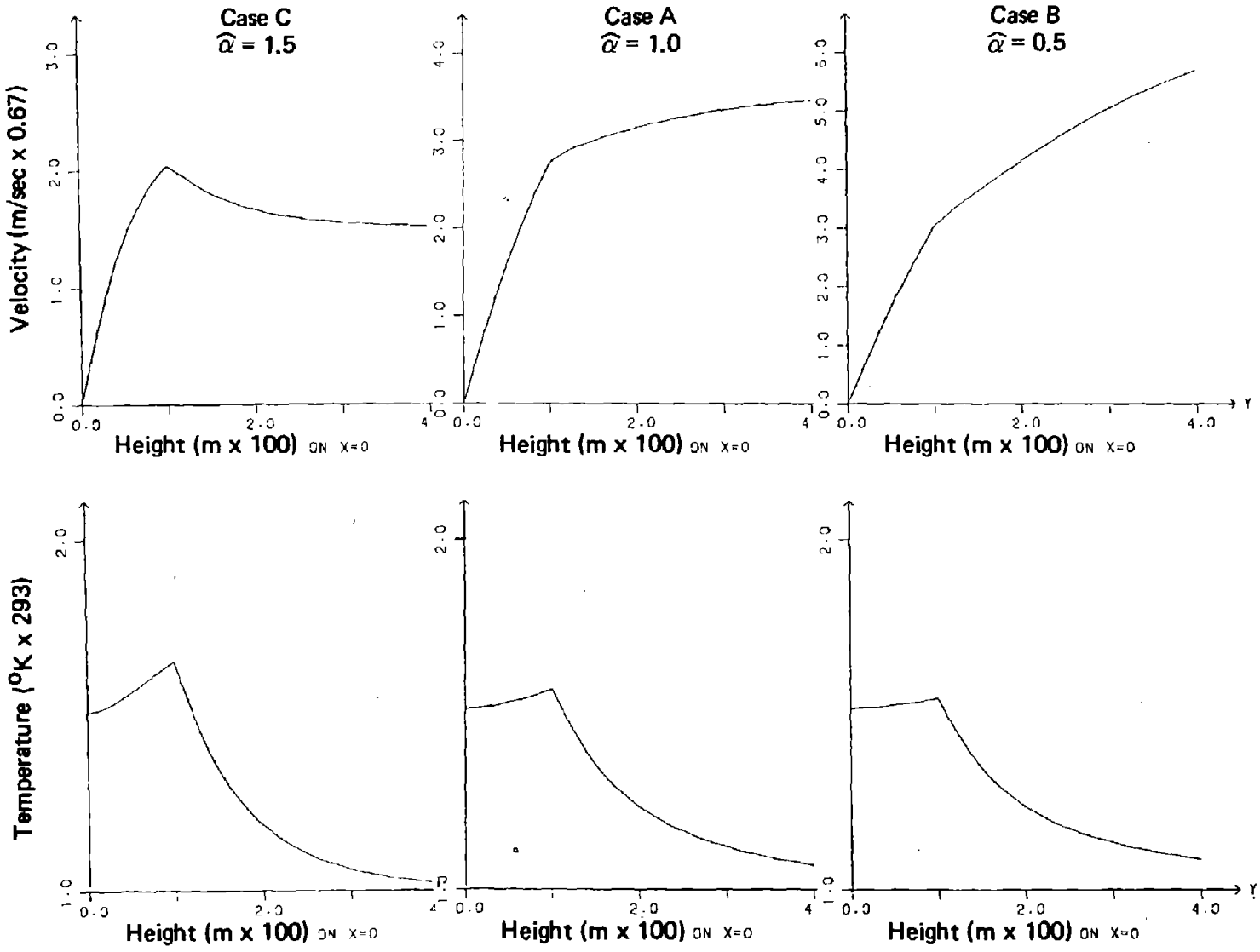


Fig. 10--Center-line velocities and temperatures generated for different inlet velocity magnitudes

the relatively longer fluid residence time in the burning region, whereas the more rapid decrease in temperature above the combustion zone indicates a higher radiative energy loss. In all cases, the temperature decreases rapidly above the burning region.

Figure 11 compares profiles of horizontal velocity as a function of the distance from the center line for the three cases, using comparable absolute values for the vectors. Velocities along the ground are similar in all three cases. However, the turning of the horizontal velocity to a vertical flow above the surface is much more rapid in the low-vertical-velocity case C than in the high-velocity case B. This reflects the interrelationship between the rising buoyant flow and the requirement for inflow air at the base of the column or in the fire region. When the column is rising vigorously (as in case B), the persistence of inward motion well above the surface is evident.

Figure 12 compares vertical velocities for the three cases as a function of horizontal range and altitude. The scale for each plot is expanded, so the plots do not have a single velocity amplitude measure. The maximum velocities are in the approximate ratio 2:3:4 = C:A:B, the peak velocity of case C (at y = 100 m) being half the peak velocity of case B (at y = 200 m). The velocity across the plume does decelerate in the lowest-boundary-velocity case C, but continues to accelerate appreciably above the burn region in the highest-boundary-velocity case B. All cases show a similar distribution of velocities across the plume, reaching a maximum along the center line and decreasing toward the edge.

Tables 2 through 7 list the horizontal and vertical velocities (U, V), temperature (T), density (RHO), and pressure (P) for the bottom and uppermost rows of mesh points in each calculation (i.e., the points at the ground surface and those at six times the fire zone height, viz., 600 m). Also shown are the pressure gradient (PX), the turbulent mixing rate as defined by the second derivative of the horizontal velocity (NU * UXX), the thermal diffusion in terms of the second derivative of temperature (K * TXX), and the net heat input rate, Q(T).

A comparison of cases A, B, and C for the bottom mesh points (Tables 2, 4, and 6) shows remarkably similar temperature, density, and

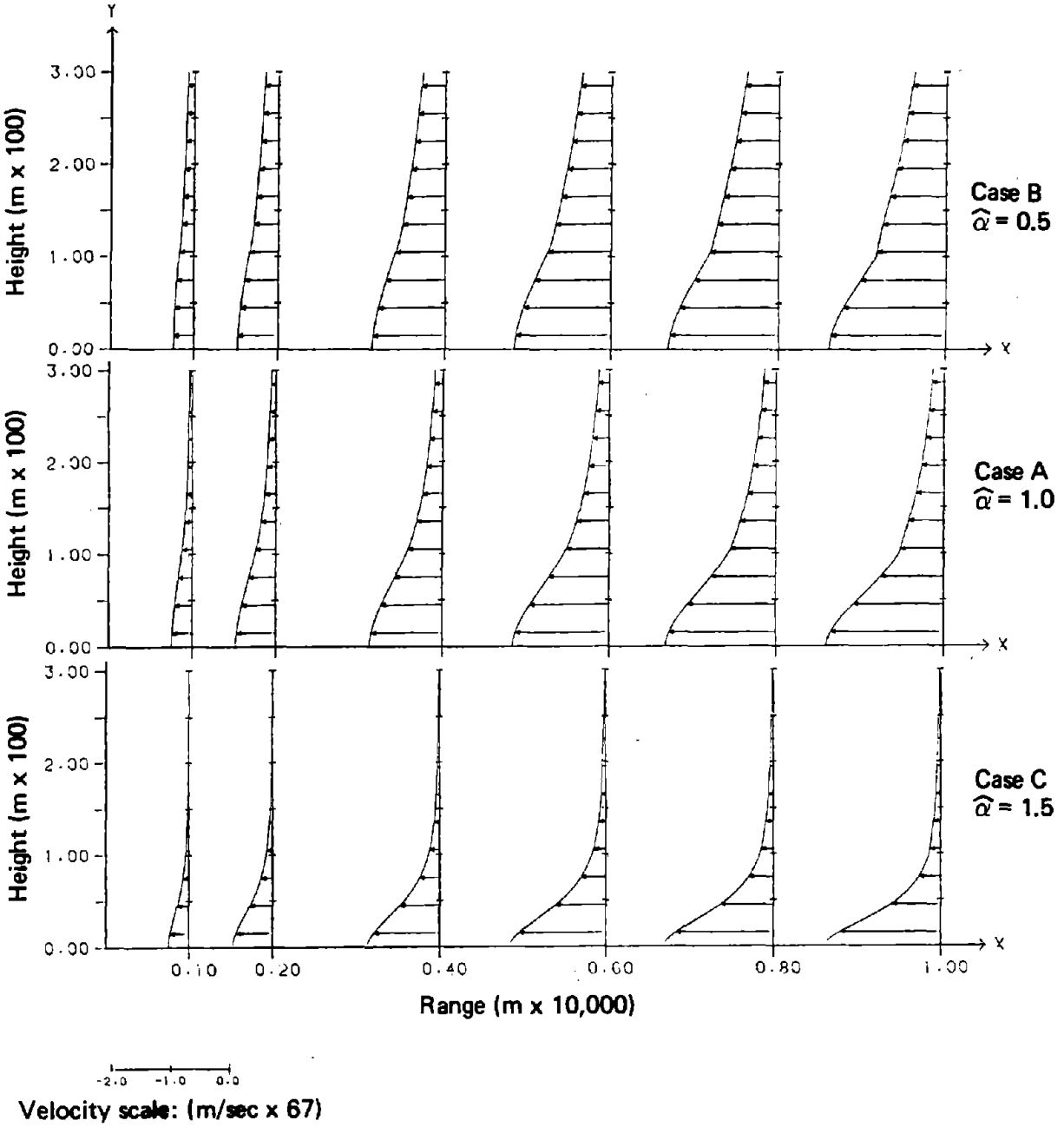


Fig. 11--Horizontal velocity profiles generated for different inlet velocity magnitudes

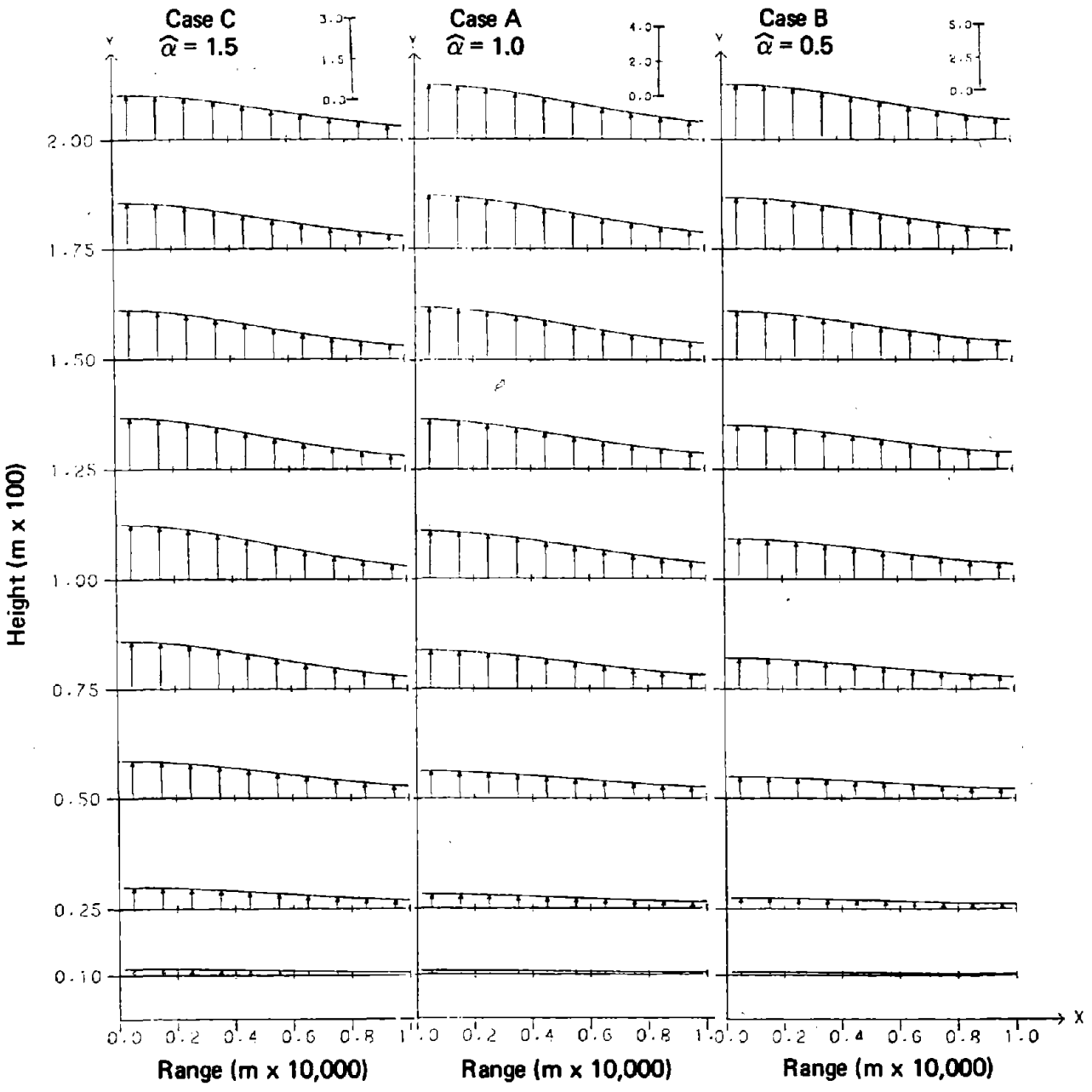


Fig. 12--Vertical velocity profiles generated for different inlet velocity magnitudes

Table 2

GROUND-LEVEL NUMERICAL PREDICTIONS FOR BASELINE PARAMETER CASE A

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: M = 40 DX = 0.025 N = 200 DY = 0.025

ITERATION PARAMETERS: ETLINE = 0.001000 MAXITL = 20 ETOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 1.000 BETA = 1.500 %LOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.000	1.516	0.660	-0.149	0.000	0.000	0.000	0.902
2	0.025	-0.084	0.000	1.516	0.660	-0.149	0.004	0.192	-0.900	0.902
3	0.050	-0.169	0.000	1.515	0.660	-0.149	0.014	0.388	-0.894	0.902
4	0.075	-0.252	0.000	1.512	0.661	-0.148	0.023	0.579	-0.884	0.903
5	0.100	-0.335	0.000	1.508	0.663	-0.147	0.031	0.764	-0.870	0.904
6	0.125	-0.417	0.000	1.505	0.665	-0.147	0.041	0.944	-0.849	0.905
7	0.150	-0.498	0.000	1.499	0.667	-0.145	0.051	1.116	-0.828	0.907
8	0.175	-0.577	0.000	1.493	0.670	-0.144	0.059	1.276	-0.800	0.909
9	0.200	-0.655	0.000	1.485	0.673	-0.142	0.067	1.426	-0.770	0.911
10	0.225	-0.731	0.000	1.477	0.677	-0.141	0.077	1.565	-0.736	0.914
11	0.250	-0.805	0.000	1.467	0.682	-0.139	0.085	1.689	-0.700	0.916
12	0.275	-0.877	0.000	1.457	0.686	-0.136	0.092	1.800	-0.661	0.919
13	0.300	-0.947	0.000	1.446	0.692	-0.134	0.102	1.896	-0.621	0.923
14	0.325	-1.015	0.000	1.434	0.697	-0.131	0.111	1.981	-0.578	0.926
15	0.350	-1.079	0.000	1.421	0.704	-0.128	0.118	2.051	-0.536	0.929
16	0.375	-1.142	0.000	1.408	0.710	-0.125	0.125	2.103	-0.492	0.933
17	0.400	-1.201	0.000	1.394	0.717	-0.122	0.132	2.142	-0.446	0.936
18	0.425	-1.258	0.000	1.380	0.725	-0.119	0.140	2.172	-0.404	0.940
19	0.450	-1.313	0.000	1.365	0.733	-0.115	0.146	2.193	-0.359	0.943
20	0.475	-1.364	0.000	1.349	0.741	-0.111	0.153	2.183	-0.314	0.947
21	0.500	-1.413	0.000	1.333	0.750	-0.108	0.161	2.174	-0.274	0.950
22	0.525	-1.459	0.000	1.317	0.759	-0.103	0.167	2.151	-0.231	0.954
23	0.550	-1.503	0.000	1.301	0.769	-0.099	0.174	2.119	-0.193	0.957
24	0.575	-1.545	0.000	1.284	0.779	-0.095	0.180	2.079	-0.153	0.960
25	0.600	-1.583	0.000	1.267	0.789	-0.090	0.185	2.027	-0.117	0.964
26	0.625	-1.619	0.000	1.250	0.800	-0.085	0.192	1.974	-0.084	0.967
27	0.650	-1.653	0.000	1.233	0.811	-0.081	0.195	1.908	-0.052	0.970
28	0.675	-1.684	0.000	1.216	0.822	-0.076	0.202	1.841	-0.021	0.973
29	0.700	-1.713	0.000	1.199	0.834	-0.071	0.209	1.772	0.005	0.976
30	0.725	-1.736	0.000	1.182	0.846	-0.065	0.211	1.693	0.033	0.978
31	0.750	-1.760	0.000	1.164	0.859	-0.060	0.215	1.616	0.054	0.981
32	0.775	-1.782	0.000	1.147	0.872	-0.055	0.222	1.539	0.077	0.983
33	0.800	-1.803	0.000	1.130	0.885	-0.049	0.228	1.460	0.095	0.985
34	0.825	-1.821	0.000	1.113	0.898	-0.043	0.230	1.377	0.111	0.988
35	0.850	-1.838	0.000	1.097	0.912	-0.037	0.234	1.298	0.128	0.990
36	0.875	-1.853	0.000	1.080	0.926	-0.031	0.242	1.223	0.140	0.992
37	0.900	-1.866	0.000	1.064	0.940	-0.025	0.246	1.145	0.151	0.994
38	0.925	-1.878	0.000	1.047	0.955	-0.019	0.248	1.067	0.161	0.995
39	0.950	-1.889	0.000	1.031	0.970	-0.014	0.252	0.996	0.169	0.997

Table 3

600-m-ALTITUDE NUMERICAL PREDICTIONS FOR BASELINE PARAMETER CASE A

FOR J = 241 (Y = 6.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	3.528	1.026	0.975	-0.960	0.000	0.000	0.000	-0.003
2	0.025	-0.001	3.524	1.026	0.975	-0.960	0.000	0.003	-0.040	-0.003
3	0.050	-0.003	3.513	1.026	0.975	-0.960	0.001	0.007	-0.039	-0.002
4	0.075	-0.004	3.497	1.026	0.975	-0.960	0.000	0.009	-0.040	-0.002
5	0.100	-0.005	3.475	1.026	0.975	-0.960	-0.001	0.011	-0.037	-0.002
6	0.125	-0.007	3.447	1.026	0.975	-0.960	0.000	0.015	-0.039	-0.002
7	0.150	-0.008	3.413	1.025	0.975	-0.960	0.001	0.019	-0.037	-0.002
8	0.175	-0.009	3.373	1.025	0.976	-0.960	0.000	0.021	-0.035	-0.002
9	0.200	-0.010	3.329	1.025	0.976	-0.960	-0.001	0.023	-0.036	-0.002
10	0.225	-0.011	3.280	1.024	0.976	-0.960	0.000	0.026	-0.035	-0.002
11	0.250	-0.013	3.226	1.024	0.977	-0.960	0.000	0.027	-0.032	-0.002
12	0.275	-0.014	3.167	1.023	0.977	-0.960	-0.001	0.028	-0.031	-0.002
13	0.300	-0.015	3.105	1.023	0.978	-0.960	0.000	0.031	-0.031	-0.002
14	0.325	-0.016	3.038	1.022	0.978	-0.960	0.001	0.034	-0.030	-0.002
15	0.350	-0.017	2.968	1.022	0.979	-0.960	0.000	0.035	-0.028	-0.002
16	0.375	-0.018	2.895	1.021	0.979	-0.960	0.000	0.035	-0.028	-0.002
17	0.400	-0.019	2.819	1.021	0.980	-0.960	-0.001	0.035	-0.025	-0.002
18	0.425	-0.020	2.741	1.020	0.981	-0.960	0.000	0.037	-0.027	-0.002
19	0.450	-0.022	2.661	1.019	0.981	-0.960	0.001	0.037	-0.022	-0.002
20	0.475	-0.023	2.579	1.018	0.982	-0.960	-0.001	0.036	-0.018	-0.002
21	0.500	-0.024	2.496	1.018	0.983	-0.960	0.000	0.038	-0.020	-0.002
22	0.525	-0.025	2.411	1.017	0.983	-0.960	-0.001	0.037	-0.018	-0.002
23	0.550	-0.025	2.326	1.016	0.984	-0.960	0.001	0.038	-0.016	-0.002
24	0.575	-0.026	2.240	1.015	0.985	-0.960	0.000	0.037	-0.013	-0.001
25	0.600	-0.026	2.154	1.014	0.986	-0.960	0.000	0.036	-0.013	-0.001
26	0.625	-0.025	2.068	1.014	0.987	-0.960	0.001	0.037	-0.010	-0.001
27	0.650	-0.025	1.983	1.013	0.987	-0.960	-0.001	0.033	-0.010	-0.001
28	0.675	-0.026	1.899	1.012	0.988	-0.960	0.000	0.034	-0.009	-0.001
29	0.700	-0.026	1.815	1.011	0.989	-0.960	0.001	0.034	-0.006	-0.001
30	0.725	-0.026	1.733	1.010	0.990	-0.960	-0.001	0.031	-0.006	-0.001
31	0.750	-0.027	1.652	1.009	0.991	-0.960	-0.002	0.029	-0.007	-0.001
32	0.775	-0.027	1.573	1.008	0.992	-0.960	0.000	0.029	-0.005	-0.001
33	0.800	-0.027	1.495	1.007	0.993	-0.960	0.001	0.030	-0.005	-0.001
34	0.825	-0.027	1.419	1.006	0.994	-0.960	-0.001	0.026	-0.002	-0.001
35	0.850	-0.027	1.346	1.006	0.994	-0.960	-0.002	0.024	-0.003	-0.001
36	0.875	-0.028	1.275	1.005	0.995	-0.960	0.002	0.027	-0.001	0.000
37	0.900	-0.028	1.205	1.004	0.996	-0.960	0.002	0.025	-0.002	0.000
38	0.925	-0.028	1.138	1.003	0.997	-0.960	-0.001	0.021	0.002	0.000
39	0.950	-0.028	1.074	1.002	0.998	-0.960	-0.001	0.019	-0.001	0.000
40	0.975	-0.028	1.012	1.001	0.999	-0.960	0.000	0.025	0.001	0.000
41	1.000	-0.028	0.981	1.000	1.000	-0.960	0.002	0.025	0.001	0.000

Table 4

GROUND-LEVEL NUMERICAL PREDICTIONS FOR PARAMETER CASE B (INCREASED MAGNITUDE OF INLET VERTICAL VELOCITY)

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: M = 40 DX = 0.025 N = 200 DY = 0.025

ITERATION PARAMETERS: ETLIME = 0.001000 MAXITL = 20 ETOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 1.500 BETA = 1.500 BLOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.000	1.516	0.660	-0.156	0.000	0.000	0.000	0.902
2	0.025	-0.045	0.000	1.516	0.660	-0.156	0.006	0.195	-0.900	0.902
3	0.050	-0.169	0.000	1.514	0.660	-0.156	0.017	0.393	-0.895	0.902
4	0.075	-0.253	0.000	1.512	0.661	-0.155	0.028	0.587	-0.883	0.903
5	0.100	-0.336	0.000	1.509	0.663	-0.154	0.039	0.776	-0.869	0.904
6	0.125	-0.418	0.000	1.506	0.665	-0.153	0.049	0.958	-0.851	0.905
7	0.150	-0.499	0.000	1.499	0.667	-0.152	0.060	1.131	-0.826	0.907
8	0.175	-0.579	0.000	1.492	0.670	-0.150	0.071	1.294	-0.800	0.909
9	0.200	-0.657	0.000	1.485	0.673	-0.148	0.081	1.445	-0.768	0.911
10	0.225	-0.733	0.000	1.476	0.677	-0.146	0.091	1.585	-0.737	0.914
11	0.250	-0.808	0.000	1.467	0.682	-0.144	0.101	1.711	-0.699	0.916
12	0.275	-0.880	0.000	1.457	0.687	-0.141	0.110	1.823	-0.661	0.919
13	0.300	-0.949	0.000	1.446	0.692	-0.138	0.119	1.922	-0.620	0.923
14	0.325	-1.017	0.000	1.434	0.698	-0.135	0.128	2.005	-0.578	0.926
15	0.350	-1.082	0.000	1.421	0.704	-0.132	0.137	2.072	-0.536	0.929
16	0.375	-1.144	0.000	1.408	0.710	-0.128	0.145	2.127	-0.491	0.933
17	0.400	-1.204	0.000	1.394	0.717	-0.124	0.152	2.165	-0.446	0.936
18	0.425	-1.261	0.000	1.379	0.725	-0.121	0.160	2.190	-0.403	0.940
19	0.450	-1.315	0.000	1.364	0.733	-0.116	0.167	2.203	-0.358	0.943
20	0.475	-1.366	0.000	1.349	0.741	-0.112	0.173	2.201	-0.315	0.947
21	0.500	-1.415	0.000	1.333	0.750	-0.108	0.180	2.188	-0.272	0.950
22	0.525	-1.461	0.000	1.317	0.759	-0.103	0.185	2.164	-0.232	0.954
23	0.550	-1.504	0.000	1.301	0.769	-0.099	0.191	2.130	-0.191	0.957
24	0.575	-1.545	0.000	1.284	0.779	-0.094	0.196	2.084	-0.154	0.960
25	0.600	-1.583	0.000	1.267	0.789	-0.089	0.200	2.034	-0.117	0.964
26	0.625	-1.618	0.000	1.250	0.800	-0.084	0.205	1.975	-0.084	0.967
27	0.650	-1.651	0.000	1.233	0.811	-0.079	0.208	1.910	-0.051	0.970
28	0.675	-1.682	0.000	1.216	0.822	-0.073	0.212	1.839	-0.022	0.973
29	0.700	-1.710	0.000	1.199	0.834	-0.068	0.215	1.766	0.005	0.976
30	0.725	-1.736	0.000	1.181	0.846	-0.063	0.218	1.688	0.031	0.978
31	0.750	-1.760	0.000	1.164	0.859	-0.057	0.220	1.608	0.055	0.981
32	0.775	-1.782	0.000	1.147	0.872	-0.051	0.223	1.527	0.076	0.983
33	0.800	-1.802	0.000	1.130	0.885	-0.046	0.225	1.445	0.095	0.985
34	0.825	-1.820	0.000	1.113	0.898	-0.040	0.226	1.363	0.111	0.988
35	0.850	-1.837	0.000	1.097	0.912	-0.035	0.227	1.281	0.127	0.990
36	0.875	-1.852	0.000	1.080	0.926	-0.029	0.229	1.202	0.140	0.992
37	0.900	-1.865	0.000	1.064	0.940	-0.023	0.230	1.122	0.150	0.994
38	0.925	-1.877	0.000	1.047	0.955	-0.017	0.231	1.047	0.161	0.995
39	0.950	-1.888	0.000	1.031	0.970	-0.012	0.231	0.971	0.167	0.997
40	0.975	-1.897	0.000	1.015	0.985	-0.006	0.232	0.900	0.175	0.999
41	1.000	-1.906	0.000	1.000	1.000	0.000	0.233	0.900	0.175	1.000

Table 5

600-m-ALTITUDE NUMERICAL PREDICTIONS FOR PARAMETER CASE B
(INCREASED MAGNITUDE OF INLET VERTICAL VELOCITY)

FOR J = 241 (Y = 6.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	6.453	1.045	0.957	-0.960	0.000	0.000	0.000	-0.004
2	0.025	-0.006	6.446	1.045	0.957	-0.960	0.000	0.023	-0.077	-0.004
3	0.050	-0.016	6.425	1.045	0.957	-0.960	0.000	0.045	-0.077	-0.004
4	0.075	-0.025	6.392	1.045	0.957	-0.960	0.000	0.067	-0.076	-0.004
5	0.100	-0.033	6.347	1.045	0.957	-0.960	0.000	0.088	-0.075	-0.004
6	0.125	-0.041	6.290	1.044	0.957	-0.960	0.000	0.108	-0.073	-0.004
7	0.150	-0.048	6.222	1.044	0.958	-0.960	0.000	0.128	-0.070	-0.004
8	0.175	-0.056	6.142	1.043	0.958	-0.960	0.000	0.146	-0.069	-0.004
9	0.200	-0.064	6.053	1.043	0.959	-0.960	0.000	0.163	-0.066	-0.004
10	0.225	-0.071	5.953	1.042	0.960	-0.960	0.000	0.179	-0.063	-0.004
11	0.250	-0.078	5.843	1.041	0.960	-0.960	0.000	0.193	-0.060	-0.004
12	0.275	-0.085	5.725	1.040	0.961	-0.960	0.000	0.205	-0.057	-0.004
13	0.300	-0.091	5.599	1.039	0.962	-0.960	0.000	0.216	-0.054	-0.004
14	0.325	-0.098	5.464	1.038	0.963	-0.960	0.000	0.225	-0.050	-0.004
15	0.350	-0.104	5.323	1.037	0.964	-0.960	0.000	0.233	-0.047	-0.004
16	0.375	-0.109	5.176	1.036	0.965	-0.960	0.000	0.238	-0.043	-0.004
17	0.400	-0.115	5.023	1.035	0.966	-0.960	0.000	0.242	-0.040	-0.003
18	0.425	-0.120	4.866	1.034	0.967	-0.960	0.000	0.245	-0.034	-0.003
19	0.450	-0.125	4.704	1.032	0.969	-0.960	0.000	0.246	-0.030	-0.003
20	0.475	-0.129	4.539	1.031	0.970	-0.960	0.000	0.245	-0.028	-0.003
21	0.500	-0.133	4.372	1.030	0.971	-0.960	0.000	0.244	-0.024	-0.003
22	0.525	-0.137	4.204	1.029	0.972	-0.960	0.000	0.241	-0.024	-0.003
23	0.550	-0.141	4.034	1.027	0.974	-0.960	0.000	0.237	-0.017	-0.003
24	0.575	-0.144	3.864	1.025	0.975	-0.960	0.000	0.231	-0.018	-0.002
25	0.600	-0.147	3.694	1.024	0.977	-0.960	0.000	0.225	-0.011	-0.002
26	0.625	-0.150	3.525	1.023	0.978	-0.960	0.000	0.219	-0.011	-0.002
27	0.650	-0.152	3.359	1.021	0.979	-0.960	0.000	0.211	-0.007	-0.002
28	0.675	-0.155	3.194	1.019	0.981	-0.960	0.000	0.203	-0.006	-0.002
29	0.700	-0.157	3.032	1.018	0.982	-0.960	0.000	0.194	-0.004	-0.002
30	0.725	-0.158	2.873	1.016	0.984	-0.960	0.000	0.185	0.000	-0.002
31	0.750	-0.160	2.718	1.015	0.985	-0.960	0.000	0.176	-0.001	-0.001
32	0.775	-0.161	2.567	1.013	0.987	-0.960	0.000	0.167	0.003	-0.001
33	0.800	-0.162	2.420	1.012	0.988	-0.960	0.000	0.158	0.002	-0.001
34	0.825	-0.163	2.278	1.010	0.990	-0.960	0.000	0.149	0.005	-0.001
35	0.850	-0.163	2.141	1.009	0.991	-0.960	0.000	0.139	0.005	-0.001
36	0.875	-0.164	2.009	1.007	0.993	-0.960	0.000	0.131	0.006	-0.001
37	0.900	-0.164	1.883	1.006	0.994	-0.960	0.000	0.122	0.009	-0.001
38	0.925	-0.164	1.762	1.004	0.996	-0.960	0.000	0.113	0.005	-0.000
39	0.950	-0.164	1.646	1.003	0.997	-0.960	0.000	0.105	0.011	0.000
40	0.975	-0.164	1.535	1.001	0.999	-0.960	0.000	0.093	0.008	0.000
41	1.000	-0.164	1.481	1.000	1.000	-0.960	0.000	0.093	0.008	0.000

Table 6

GROUND-LEVEL NUMERICAL PREDICTIONS FOR PARAMETER CASE C (DECREASED MAGNITUDE OF INLET VERTICAL VELOCITY)

FIRE: SOLUTION OF COMBUSTION LAYER BOUNDARY VALUE PROBLEM

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: N = 40 DX = 0.025 N = 200 DY = 0.025

ITERATION PARAMETERS: EILIMI = 0.001000 MAXITL = 20 EIOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 0.500 BETA = 1.500 BLOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(I)
1	0.000	0.000	0.000	1.504	0.663	-0.125	0.000	0.000	0.000	0.904
2	0.025	-0.088	0.000	1.504	0.663	-0.125	0.003	0.207	-0.903	0.904
3	0.050	-0.175	0.000	1.505	0.664	-0.125	0.010	0.416	-0.896	0.905
4	0.075	-0.262	0.000	1.504	0.665	-0.124	0.016	0.620	-0.887	0.905
5	0.100	-0.349	0.000	1.501	0.666	-0.124	0.022	0.819	-0.870	0.906
6	0.125	-0.434	0.000	1.496	0.668	-0.123	0.029	1.009	-0.850	0.908
7	0.150	-0.518	0.000	1.491	0.671	-0.122	0.035	1.191	-0.826	0.909
8	0.175	-0.600	0.000	1.484	0.674	-0.121	0.041	1.362	-0.797	0.911
9	0.200	-0.681	0.000	1.477	0.677	-0.120	0.048	1.519	-0.766	0.914
10	0.225	-0.760	0.000	1.468	0.681	-0.119	0.054	1.665	-0.730	0.916
11	0.250	-0.837	0.000	1.459	0.685	-0.118	0.060	1.795	-0.694	0.919
12	0.275	-0.911	0.000	1.449	0.690	-0.116	0.066	1.911	-0.653	0.922
13	0.300	-0.983	0.000	1.438	0.696	-0.114	0.073	2.013	-0.611	0.925
14	0.325	-1.053	0.000	1.426	0.701	-0.112	0.079	2.096	-0.567	0.928
15	0.350	-1.120	0.000	1.413	0.708	-0.110	0.085	2.167	-0.523	0.931
16	0.375	-1.184	0.000	1.400	0.714	-0.108	0.091	2.219	-0.478	0.935
17	0.400	-1.246	0.000	1.386	0.721	-0.106	0.097	2.258	-0.431	0.938
18	0.425	-1.304	0.000	1.372	0.729	-0.103	0.103	2.281	-0.387	0.942
19	0.450	-1.360	0.000	1.357	0.737	-0.101	0.109	2.291	-0.342	0.945
20	0.475	-1.413	0.000	1.342	0.745	-0.098	0.115	2.287	-0.298	0.948
21	0.500	-1.463	0.000	1.326	0.754	-0.095	0.121	2.271	-0.255	0.952
22	0.525	-1.511	0.000	1.310	0.763	-0.092	0.127	2.244	-0.213	0.955
23	0.550	-1.555	0.000	1.294	0.773	-0.089	0.134	2.206	-0.174	0.959
24	0.575	-1.597	0.000	1.277	0.783	-0.085	0.140	2.158	-0.136	0.962
25	0.600	-1.636	0.000	1.261	0.793	-0.082	0.146	2.103	-0.099	0.965
26	0.625	-1.672	0.000	1.244	0.804	-0.078	0.152	2.041	-0.067	0.968
27	0.650	-1.706	0.000	1.227	0.815	-0.074	0.158	1.972	-0.034	0.971
28	0.675	-1.737	0.000	1.210	0.826	-0.070	0.165	1.898	-0.006	0.974
29	0.700	-1.766	0.000	1.194	0.838	-0.066	0.172	1.822	0.021	0.976
30	0.725	-1.793	0.000	1.177	0.850	-0.061	0.178	1.742	0.045	0.979
31	0.750	-1.818	0.000	1.160	0.862	-0.057	0.185	1.659	0.069	0.981
32	0.775	-1.840	0.000	1.143	0.875	-0.052	0.193	1.577	0.088	0.984
33	0.800	-1.861	0.000	1.127	0.888	-0.047	0.200	1.493	0.105	0.986
34	0.825	-1.879	0.000	1.110	0.901	-0.042	0.208	1.411	0.121	0.988
35	0.850	-1.896	0.000	1.094	0.914	-0.037	0.216	1.329	0.135	0.990
36	0.875	-1.911	0.000	1.078	0.928	-0.031	0.225	1.249	0.146	0.992
37	0.900	-1.925	0.000	1.062	0.942	-0.025	0.234	1.172	0.156	0.994
38	0.925	-1.937	0.000	1.046	0.956	-0.020	0.244	1.096	0.164	0.995
39	0.950	-1.948	0.000	1.030	0.970	-0.013	0.254	1.025	0.171	0.997
40	0.975	-1.958	0.000	1.015	0.985	-0.007	0.266	0.954	0.176	0.999
41	1.000	-1.966	0.000	1.000	1.000	0.000	0.272	0.954	0.176	1.000

Table 7

600-m-ALTITUDE NUMERICAL PREDICTIONS FOR PARAMETER CASE C
(DECREASED MAGNITUDE OF INLET VERTICAL VELOCITY)

FOR J = 21 (Y = 6.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(I)
1	0.000	0.000	1.507	1.002	0.998	-0.960	0.000	0.000	0.000	0.000
2	0.025	0.000	1.505	1.002	0.998	-0.960	0.000	0.000	-0.004	0.000
3	0.050	0.000	1.500	1.002	0.998	-0.960	0.000	0.000	-0.003	0.000
4	0.075	0.000	1.493	1.002	0.998	-0.960	0.000	0.000	-0.004	0.000
5	0.100	0.000	1.484	1.002	0.998	-0.960	0.000	0.000	-0.005	0.000
6	0.125	0.000	1.472	1.002	0.998	-0.960	0.000	0.000	-0.002	0.000
7	0.150	0.000	1.457	1.002	0.998	-0.960	0.000	0.001	-0.005	0.000
8	0.175	0.000	1.441	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
9	0.200	0.000	1.422	1.002	0.998	-0.960	0.000	0.001	-0.003	0.000
10	0.225	0.000	1.401	1.002	0.998	-0.960	0.000	0.001	-0.004	0.000
11	0.250	0.000	1.378	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
12	0.275	0.000	1.354	1.002	0.998	-0.960	0.000	0.001	-0.003	0.000
13	0.300	0.000	1.328	1.002	0.998	-0.960	0.000	0.001	-0.005	0.000
14	0.325	-0.001	1.300	1.002	0.998	-0.960	0.000	0.001	-0.001	0.000
15	0.350	-0.001	1.271	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
16	0.375	-0.001	1.241	1.002	0.998	-0.960	0.000	0.001	-0.003	0.000
17	0.400	-0.001	1.210	1.002	0.998	-0.960	0.000	0.001	-0.003	0.000
18	0.425	-0.001	1.177	1.002	0.998	-0.960	0.000	0.001	-0.001	0.000
19	0.450	-0.001	1.145	1.002	0.998	-0.960	0.000	0.001	-0.003	0.000
20	0.475	-0.001	1.111	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
21	0.500	-0.001	1.078	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
22	0.525	-0.001	1.043	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
23	0.550	-0.001	1.009	1.002	0.998	-0.960	0.000	0.001	-0.002	0.000
24	0.575	-0.001	0.975	1.001	0.999	-0.960	0.000	0.001	0.000	0.000
25	0.600	-0.001	0.941	1.001	0.999	-0.960	0.000	0.001	-0.003	0.000
26	0.625	-0.001	0.907	1.001	0.999	-0.960	0.000	0.001	0.001	0.000
27	0.650	-0.001	0.873	1.001	0.999	-0.960	0.000	0.001	-0.002	0.000
28	0.675	-0.001	0.840	1.001	0.999	-0.960	0.000	0.001	-0.002	0.000
29	0.700	-0.001	0.807	1.001	0.999	-0.960	0.000	0.001	0.000	0.000
30	0.725	-0.001	0.775	1.001	0.999	-0.960	0.000	0.001	-0.002	0.000
31	0.750	-0.001	0.744	1.001	0.999	-0.960	0.000	0.001	0.000	0.000
32	0.775	-0.001	0.713	1.001	0.999	-0.960	0.000	0.001	-0.001	0.000
33	0.800	-0.001	0.683	1.001	0.999	-0.960	0.000	0.001	-0.001	0.000
34	0.825	-0.001	0.654	1.001	0.999	-0.960	0.000	0.001	0.000	0.000
35	0.850	-0.001	0.625	1.001	0.999	-0.960	0.000	0.001	0.000	0.000
36	0.875	-0.001	0.597	1.000	1.000	-0.960	0.000	0.001	0.000	0.000
37	0.900	-0.001	0.570	1.000	1.000	-0.960	0.000	0.001	-0.002	0.000
38	0.925	-0.001	0.544	1.000	1.000	-0.960	0.000	0.001	0.001	0.000
39	0.950	-0.001	0.518	1.000	1.000	-0.960	0.000	0.000	0.001	0.000
40	0.975	-0.001	0.494	1.000	1.000	-0.960	0.000	-0.004	-0.002	0.000
41	1.000	-0.001	0.481	1.000	1.000	-0.960	0.000	-0.004	-0.002	0.000

pressure profiles as well as similar horizontal velocity gradients. The pressure gradient is slightly higher near the outer boundary for the case of the lowest vertical velocity (case C, Table 6). In general, all parameters show remarkably similar behavior over this wide range of boundary velocity values. Interestingly, the ratio of centerline velocity to boundary velocity at the top of the combustion layer (a height of 600 m) is highest (Table 5) for the highest-boundary-velocity case B ($\hat{\alpha}$ = 1.5). The ratio between the central and boundary velocities is nearly 4.4. For case A ($\hat{\alpha}$ = 1.0), the ratio is 3.6 (Table 3); and for the lowest-boundary-velocity case C, the ratio is 3.1 (Table 7). In all cases, the pressure gradients are essentially zero at the "top" of the combustion layer; and the diffusion terms are negligible in the lowest-velocity case (Table 7).

Cases F and G explore the influence of radiation loss. Case F represents an increase in the radiation loss rate, and case G represents a decrease (in both cases, the change is about a factor of 2). No significant changes are evident, and the results resemble those for the baseline case. The only discernible change is in the centerline temperature at the bottom and top of the combustion zone, where changes of about 1 percent occur for both increased and decreased radiation loss rates.

The influence of the diffusion terms is evident in Fig. 13, which compares enhanced turbulent diffusion with the standard case. In case H, the coefficients M_{11} and K_1 are increased by a factor of 3 above those assumed in the baseline case. The velocity vector plot shows a similar turning of the flow in the lower region, but with a much more uniform velocity across the entire column above the combustion region.

Figure 14 compares pressure and temperature contours. The increased turbulence rapidly reduces the pressure and temperature gradients in the rising column.

The temperature and velocity profiles along the center line in Fig. 15 show again the rapid drop in temperature above the fire zone and the decrease in velocity above the burning region. Clearly, the enhanced diffusion substantially lowers both the temperature and the velocity on the center line (a factor of 4 in velocity).

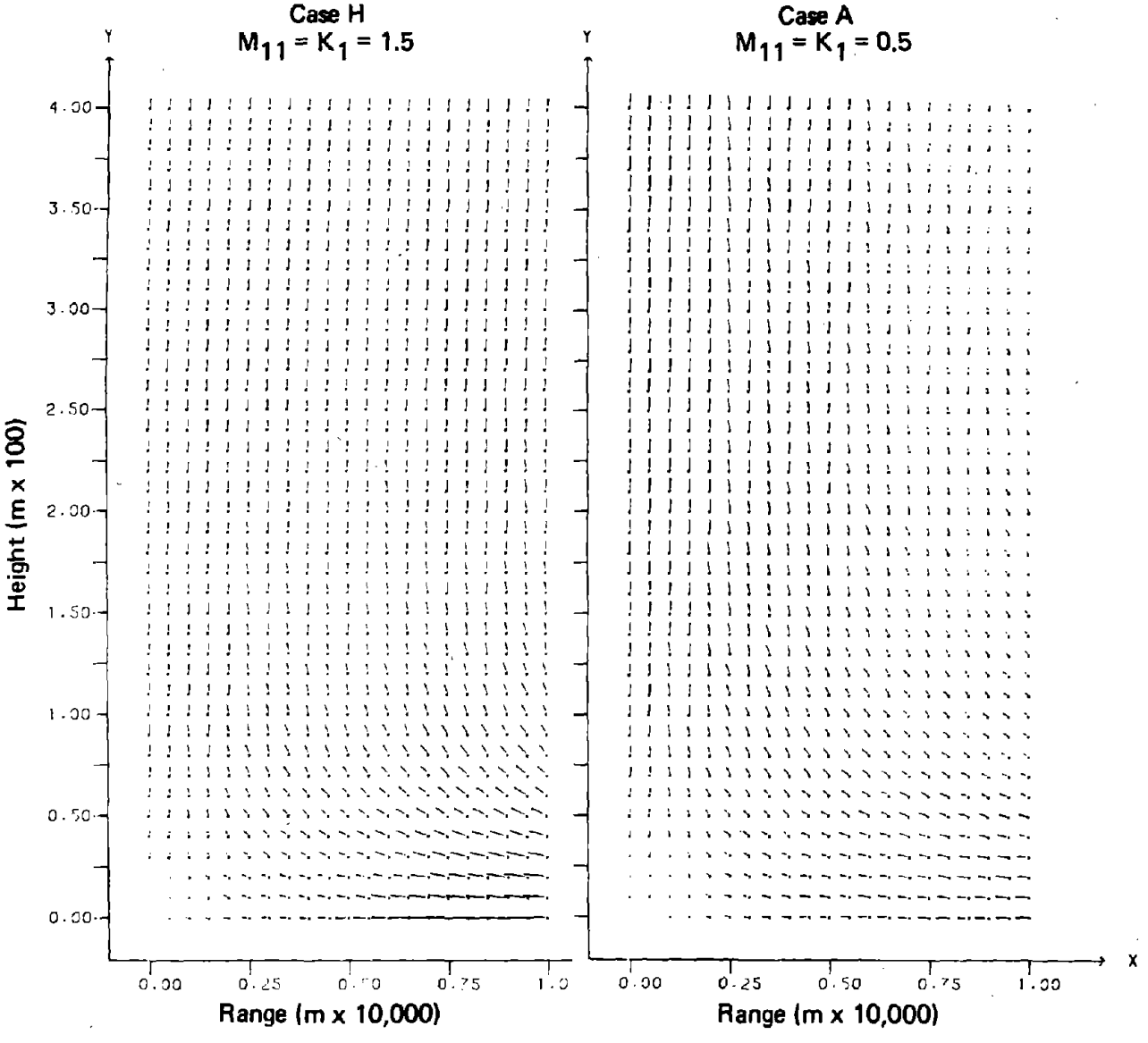


Fig. 13--Velocity fields generated for different values of common heat/momentum diffusion coefficients

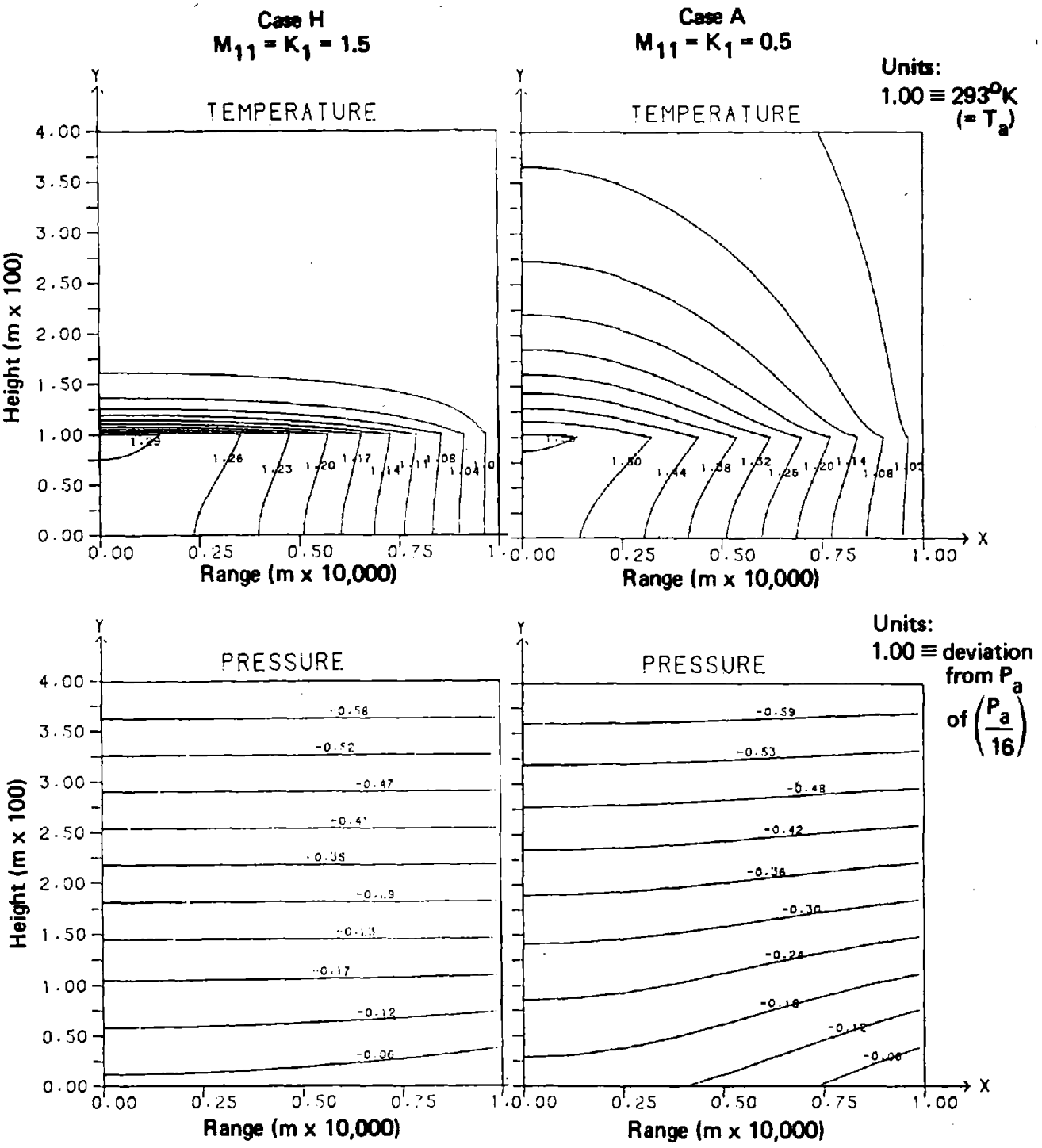


Fig. 14--Temperature and pressure contours generated for different values of common heat/momentum diffusion coefficients

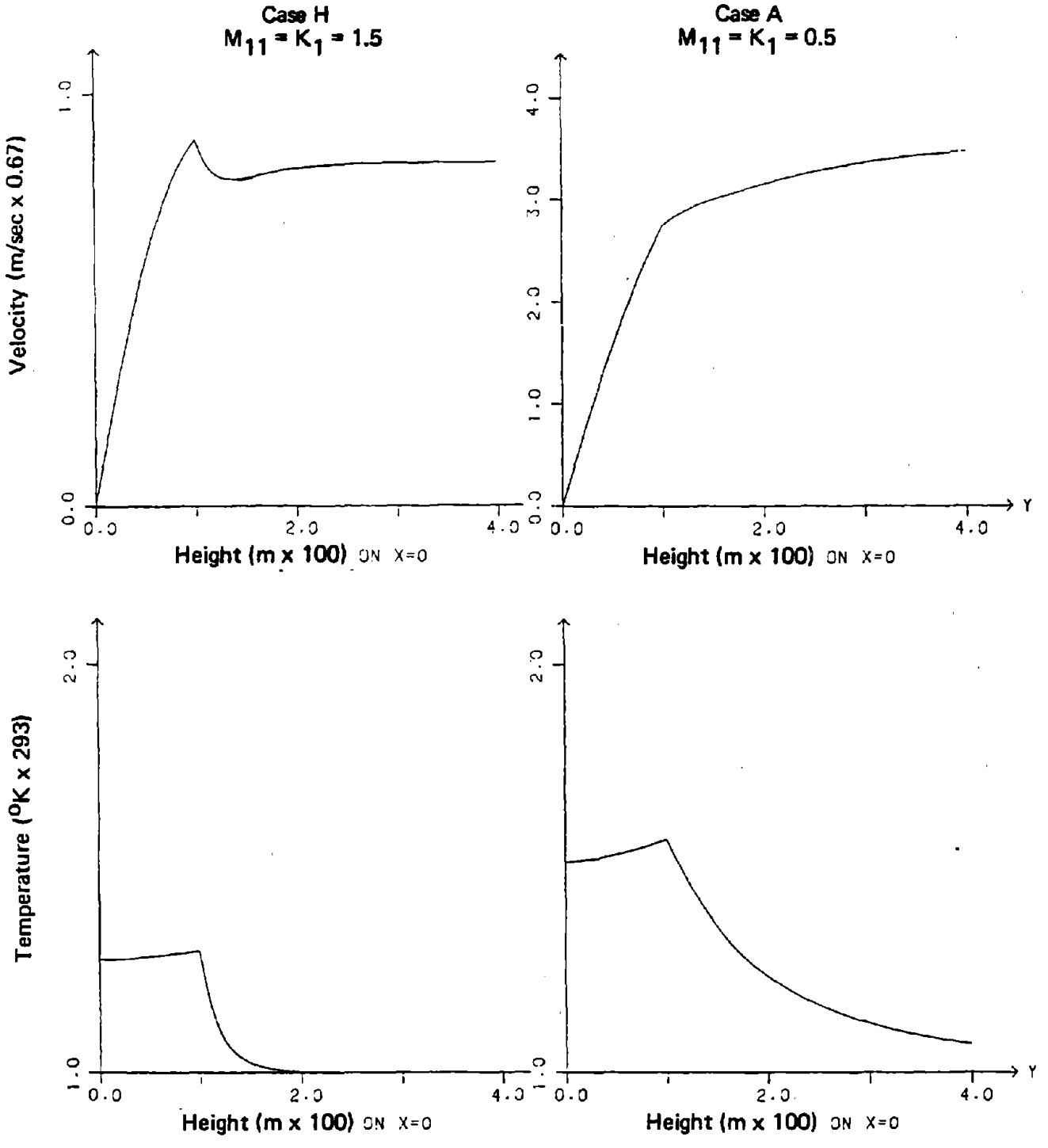


Fig. 15--Center-line velocities and temperatures generated for different values of common heat/momentum diffusion coefficients

Figure 16 shows the horizontal velocities to be of lower magnitude everywhere, and more rapidly suppressed above and even in the fire zone for the increased-turbulent-mixing case H.

Figure 17 shows for the enhanced diffusion case a very uniform velocity profile across the rising plume, i.e., virtually all the velocity is vertical and quite uniform, with approximately the value of the boundary velocity. It is slightly lower in the center of the plume owing to the dominant effect of the diffusive forces. In contrast, the baseline case A resembles a Gaussian plume distribution, as expected. It appears that the larger coefficients for diffusion may be unrealistic.

Case L displays an enhancement of the diffusion of momentum (larger M_{11}) along with a reduction in the heat diffusion coefficient K_1. A comparison with the standard case A is made in Figs. 18 through 22, illustrating the importance of balancing boundary conditions and adopting realistic physical assumptions. In this case, the assumed outer-boundary vertical velocity appears to be incompatible with the enhanced diffusion of momentum and reduced diffusion of heat in the fire and the rising column. The fact that the vertical velocities are high at the boundary of the plume, becoming smaller toward the center of the plume as shown in Fig. 18, suggests that the combination of coefficients used in case L may be incorrect. It may be unrealistic to pair such a large diffusion of momentum with correspondingly little heat dissipation.

The effect of enhanced turbulence is illustrated in Fig. 19, which compares temperature and pressure contours. Though pressure gradients are suppressed in case L, both the temperature gradient above the fire and the peak temperature are much higher. The peak temperature is nearly 2.25 (at $y = 1$) and drops to near unity before $y = 2.5$. The baseline case A has a peak of 1.56 (at $y = 1$), but does not approach unity until $y > 4$.

Similarly, as shown in Fig. 20, the center-line temperature and velocity profiles with altitude show higher temperatures in the fire and a more rapid decrease in temperature above the heated region. In the velocity comparison, we see for case L a decrease in the magnitude of the vertical velocities even in the burning region.

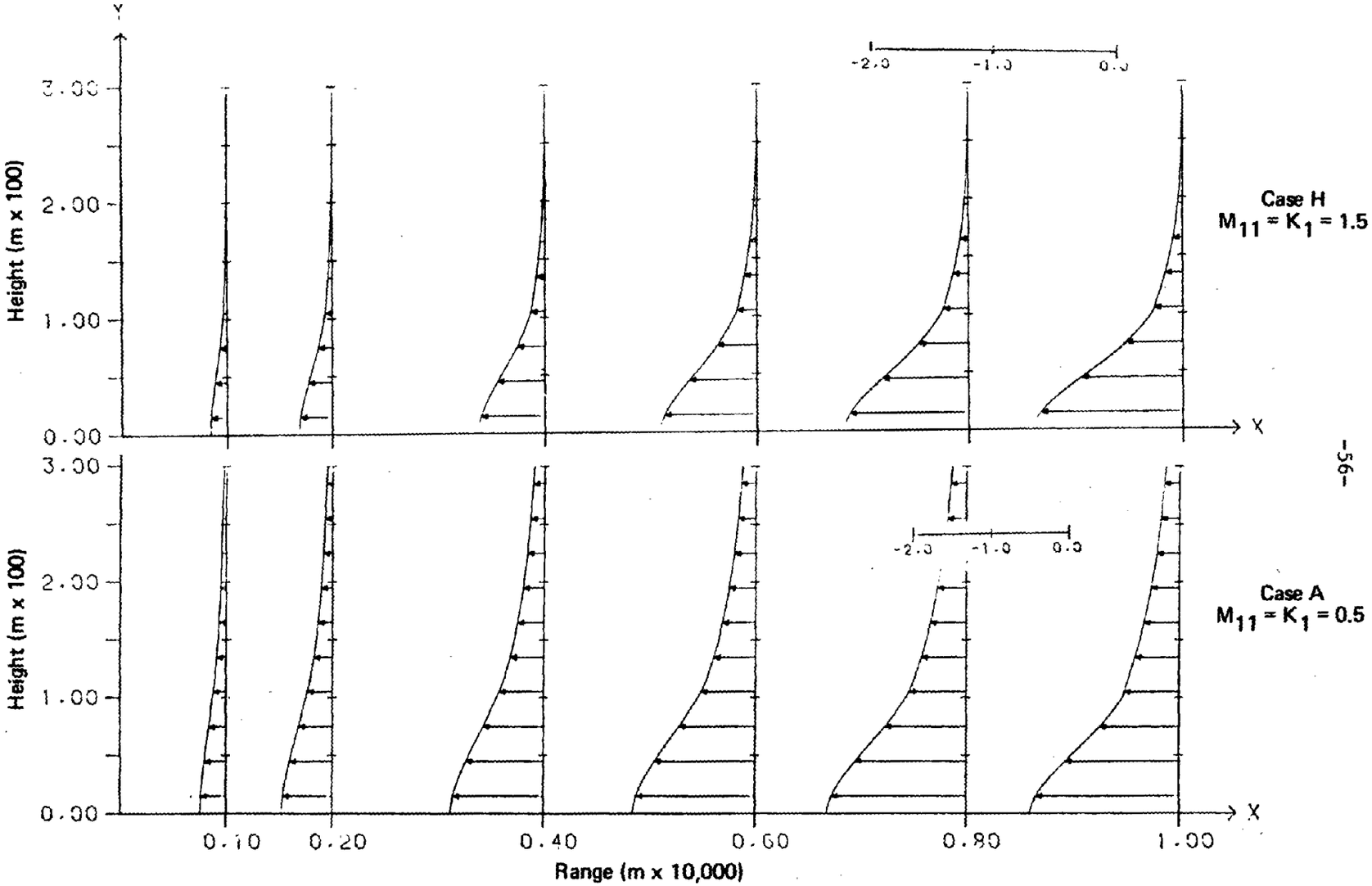


Fig. 16--Horizontal velocity profiles generated for different values of common heat/momentum diffusion coefficients

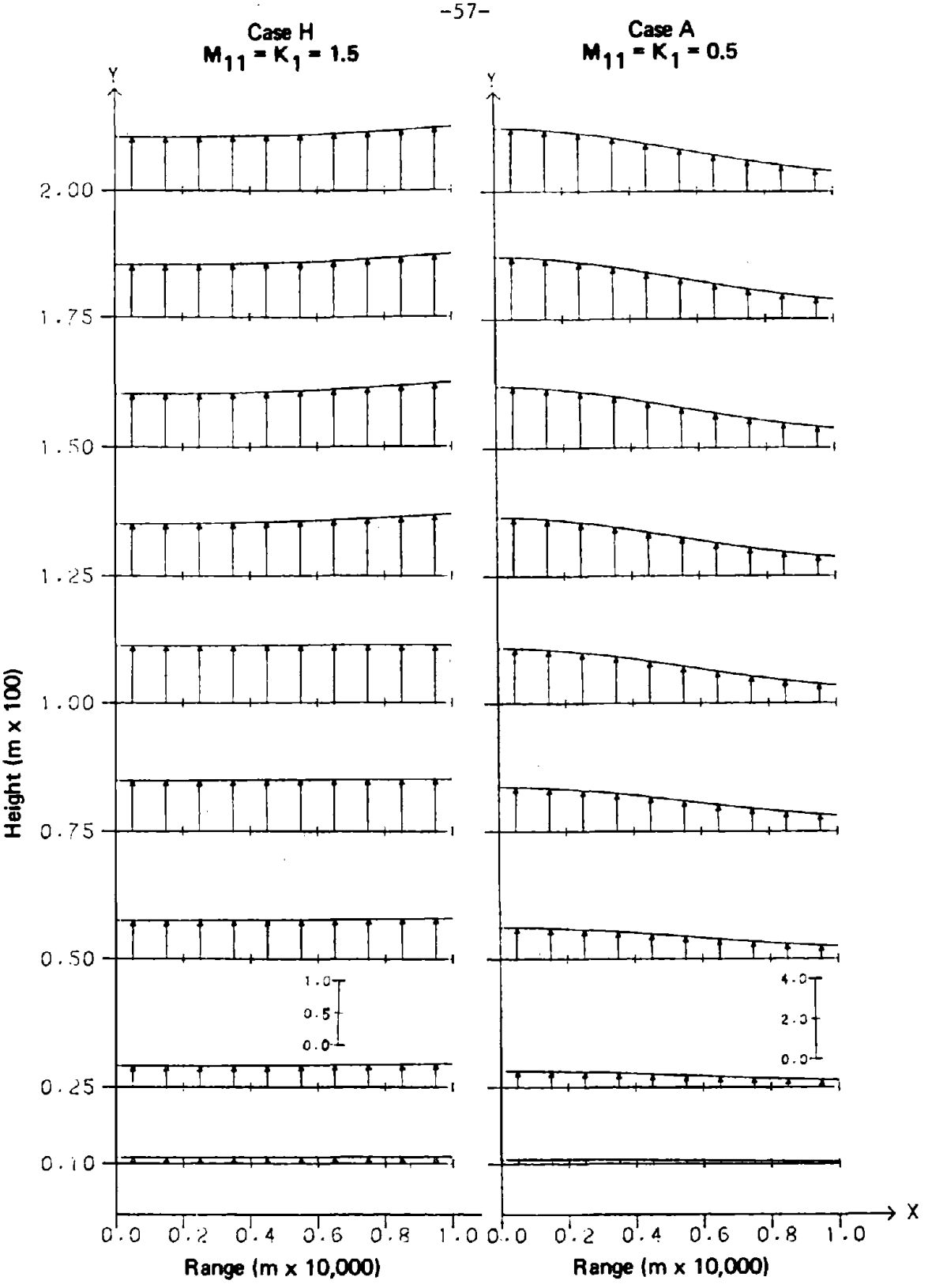


Fig. 17--Vertical velocity profiles generated for different values of common heat/momentum diffusion coefficients

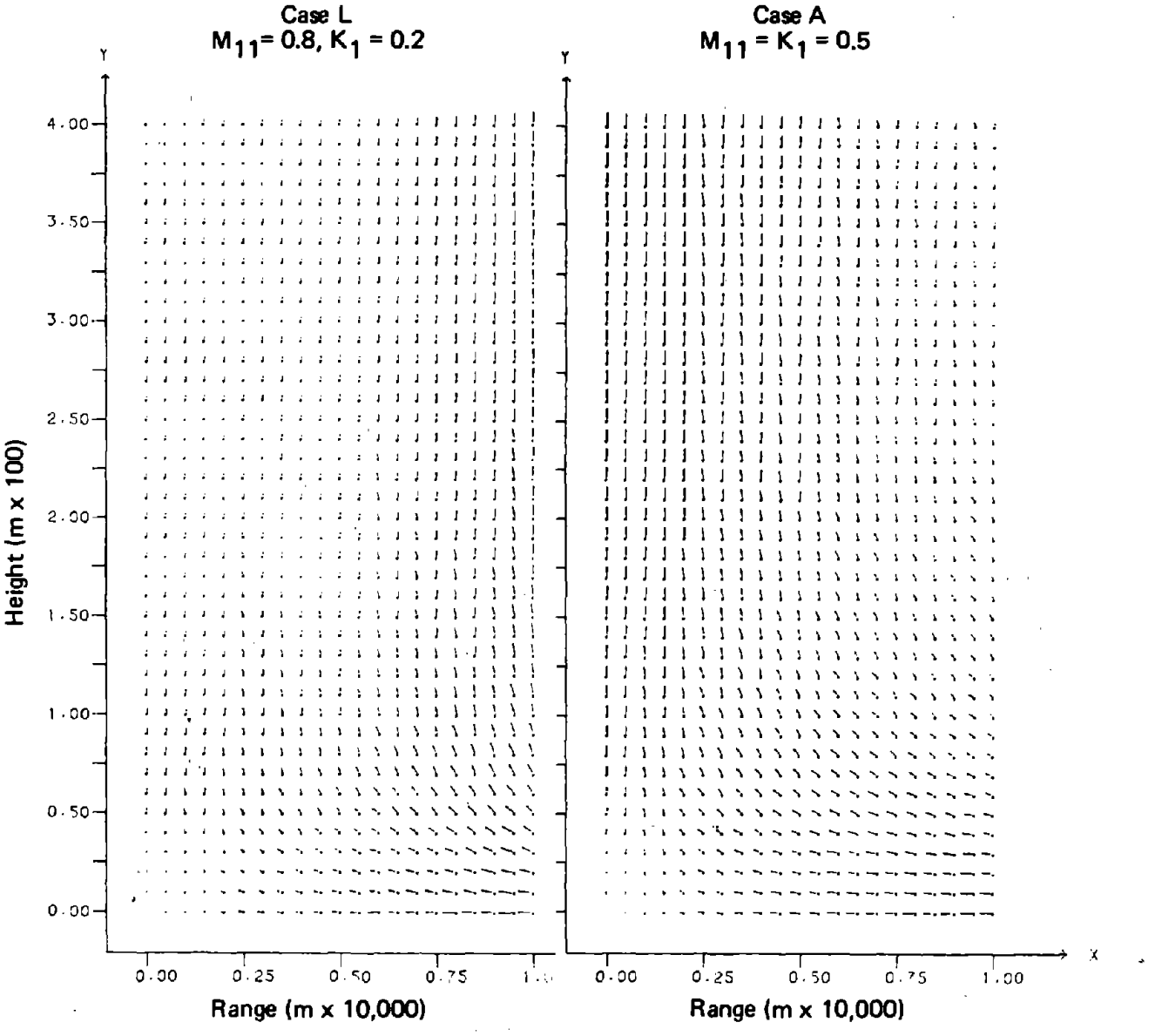


Fig. 18--Velocity fields generated for selected diffusion coefficient mixes

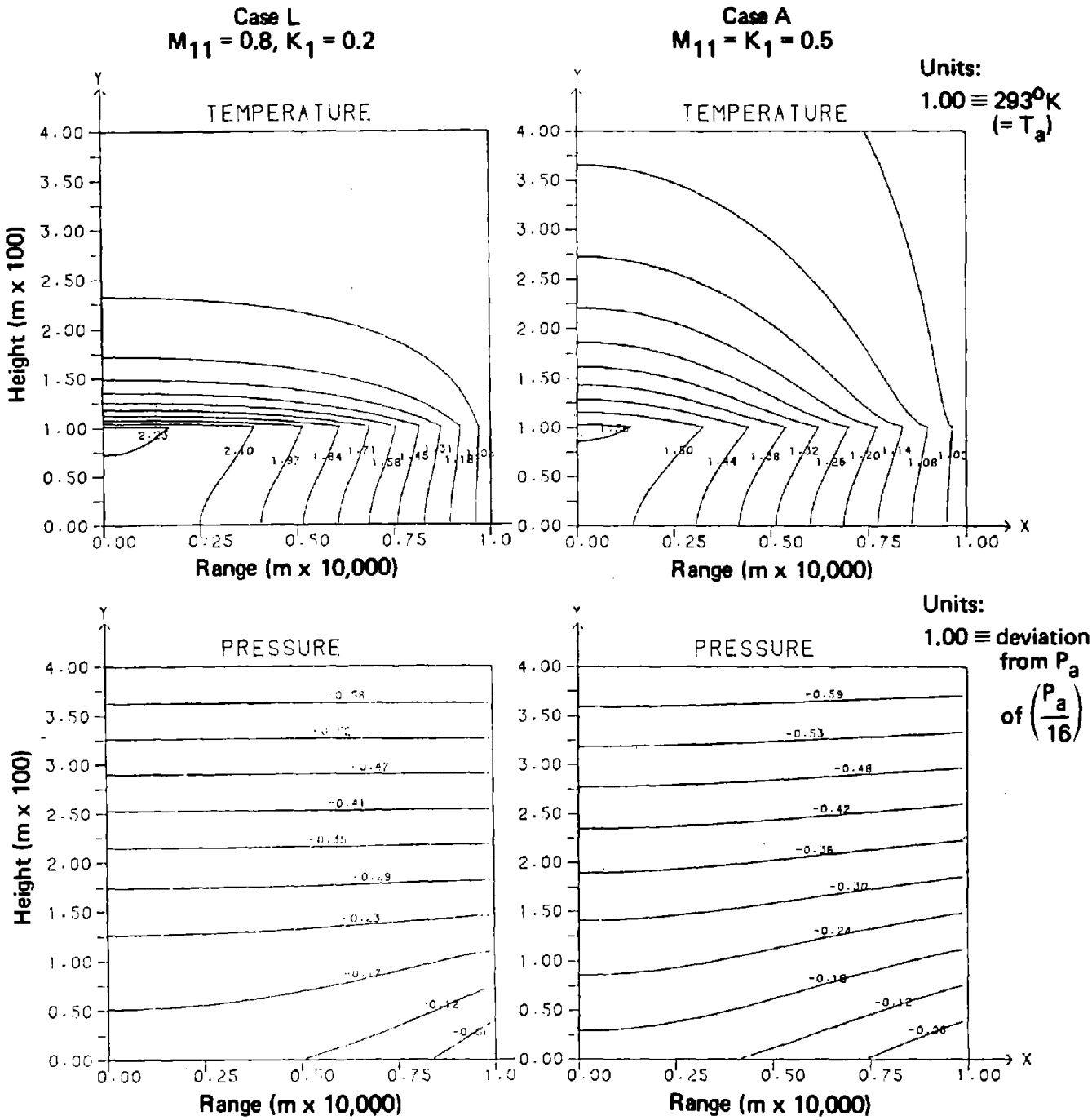


Fig. 19--Temperature and pressure contours generated for selected diffusion coefficient mixes

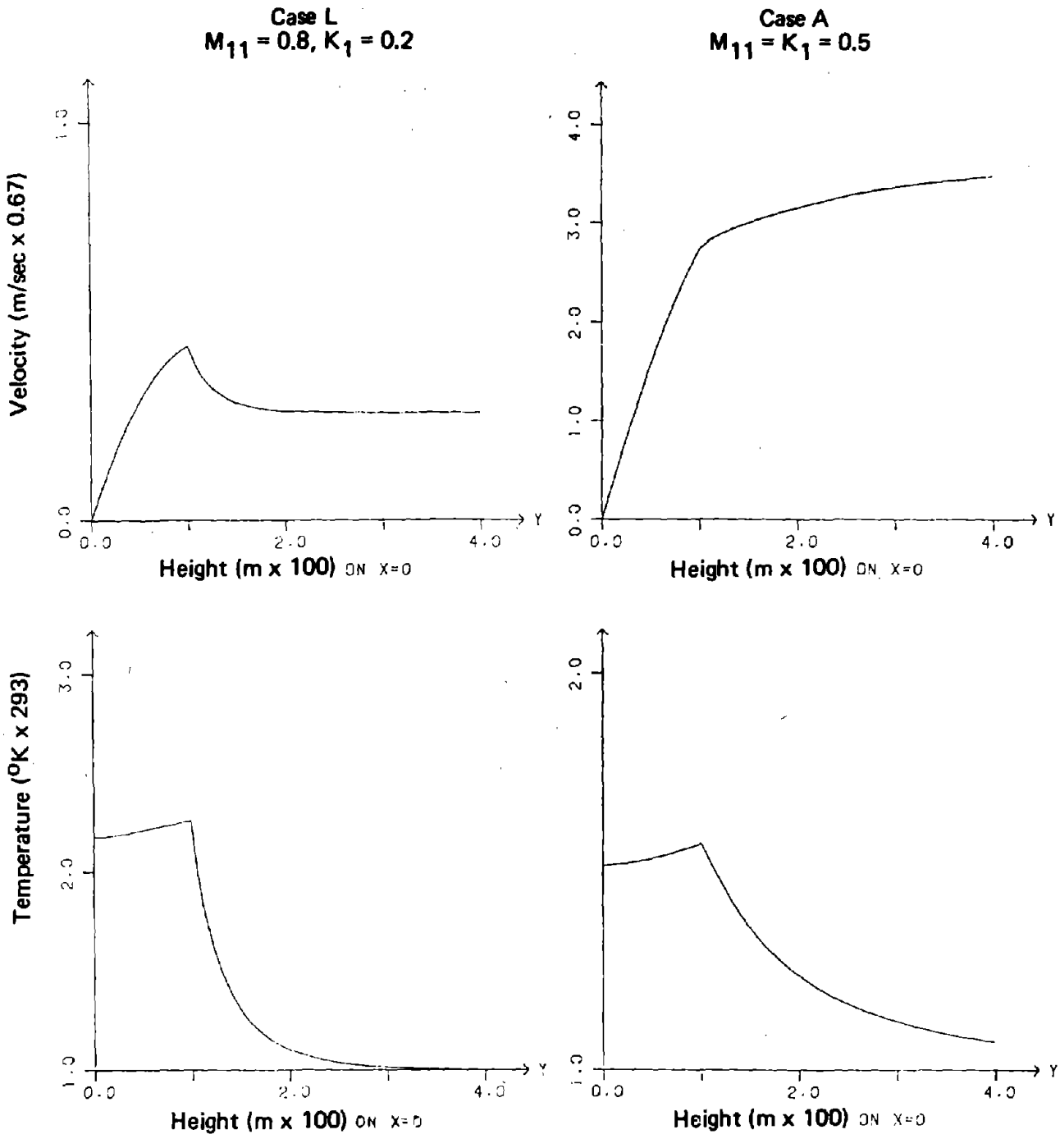
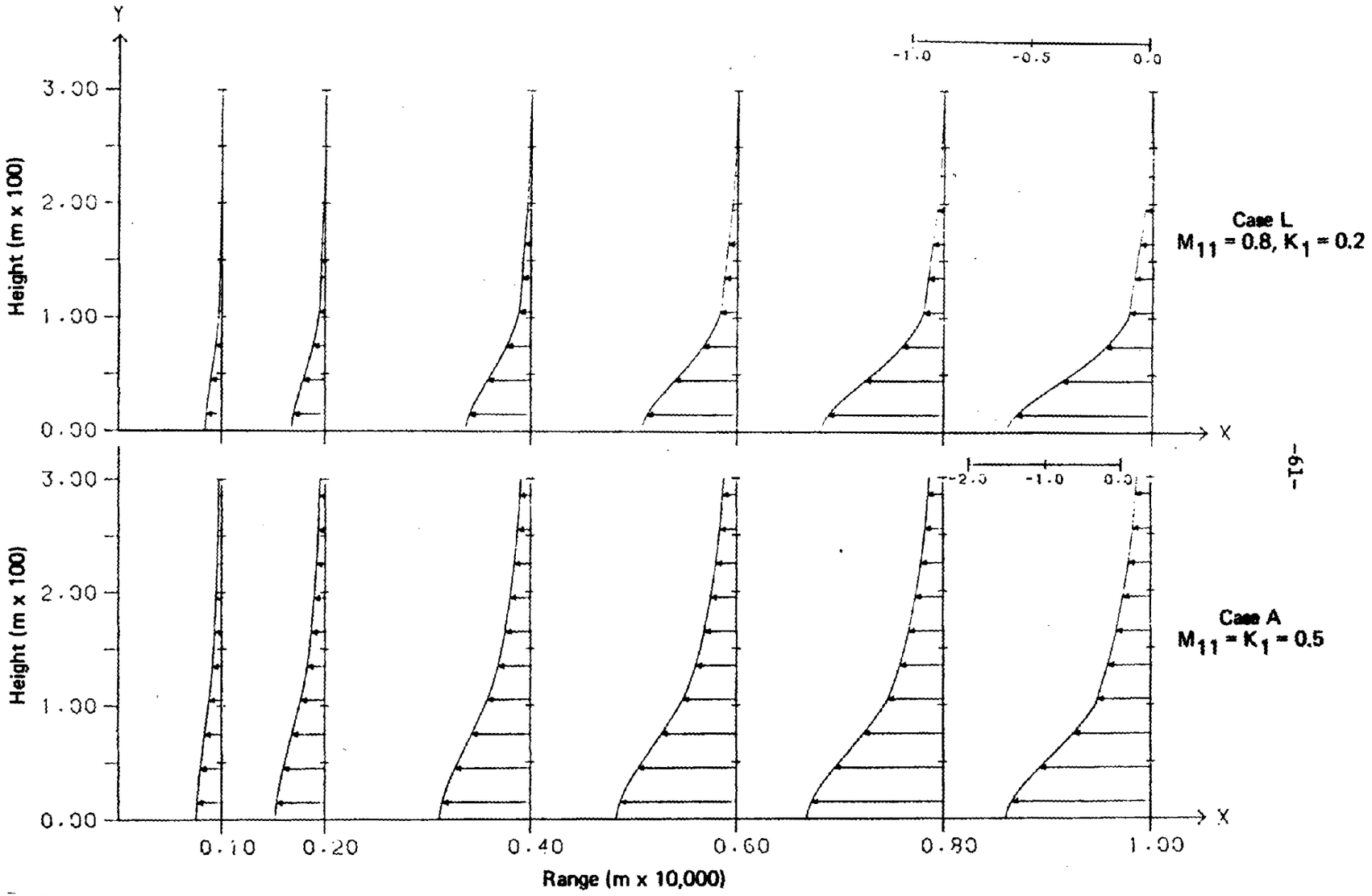


Fig. 20--Center-line velocities and temperatures generated for selected diffusion coefficient mixes



-61-

x 67)

generated for selected diffusion coefficient mixes

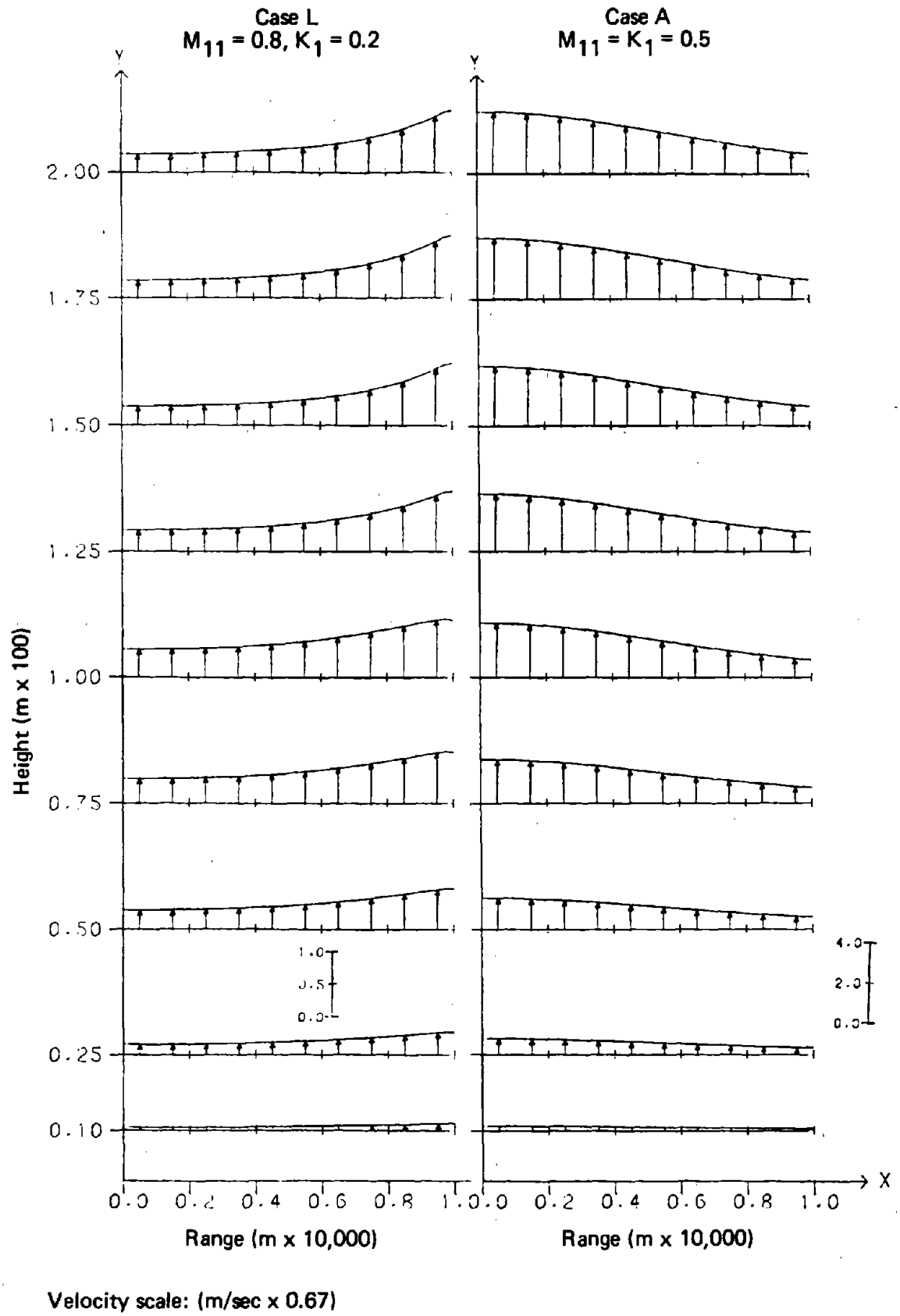


Fig. 22--Vertical velocity profiles generated for selected diffusion coefficient mixes

Figure 21 shows the effect on horizontal velocities to be not just rapid damping above the surface but also a dramatic reduction in the peak horizontal velocity as well. The peak-horizontal-velocity case L is only about a third of the corresponding peak horizontal velocity for the baseline case (note the difference in scales).

Figure 22 shows the effect of increased turbulent momentum transfer with a reversal of the vertical velocity profiles. The prescribed velocity at the outer boundary maintains the vertical flow at $x = 1$; however, the center-line velocity decreases with increasing altitude. In case A, the center-line velocity continues to increase with y.

A series of calculations were performed with variations in the combustive-heat addition rate $q(x, y)$. Figure 23 illustrates the distributions considered. Cases A through L correspond to a uniform rate of unity (in dimensionless units). Case M had double that rate, and case N had half. Cases O and P used sinusoidal distributions in horizontal range (but still uniform in height from the surface up to the top of the burning region, $y = 1$). Case O used two cycles from $x = 0$ to $x = 1$, whereas case P used ten.

Case M (Fig. 24), with double the burning rate, appears to have a velocity vector plot similar to that of case L (Fig. 18). In each case, the velocity field is suppressed along the center line with vertical velocities maintained by the boundary condition at the outer edge of the combustion region. Both cases contrast strikingly with the baseline case.

Figure 25 shows the similar suppression of the temperature and pressure contours (cf. Fig. 19). Likewise, Fig. 26 illustrates the rapid drop in temperature above the fire zone, and an even greater reduction of the velocities--again, not unlike that of case L (Fig. 20).

Figures 27 and 28 illustrate the changes in the horizontal and vertical velocities across the combustion layer, further showing the trend similar to that of the case with enhanced mixing and suppressed heat diffusion (Figs. 21 and 22). Both cases show rapidly decreasing horizontal velocities with altitude and decreasing vertical velocities along the axis. The similar behavior exhibited in cases L and M shows that either an increase in heat addition (case M: $Q = 2$, $K_1 = 0.5$)

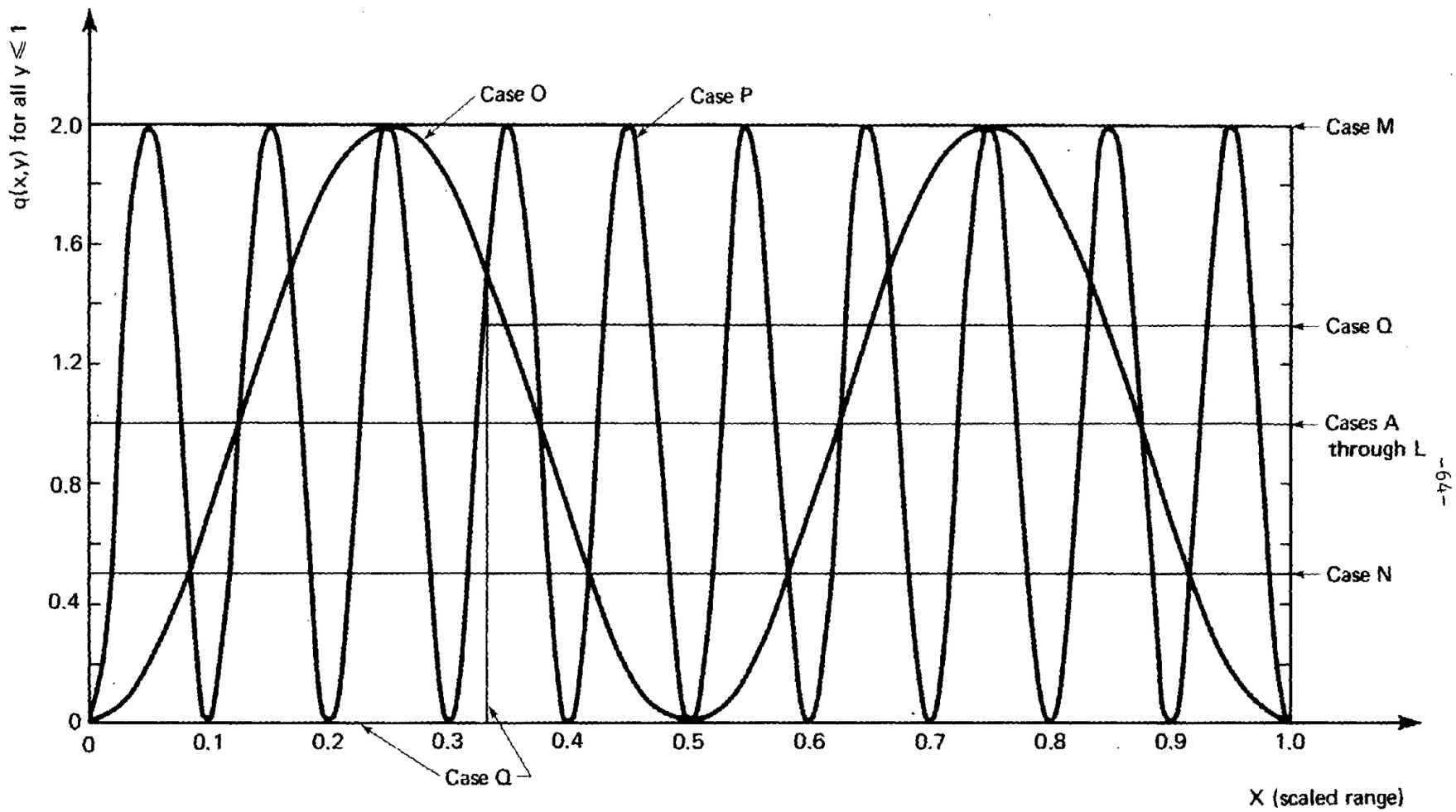


Fig. 23--Spatial distributions of heat-release-rate variations

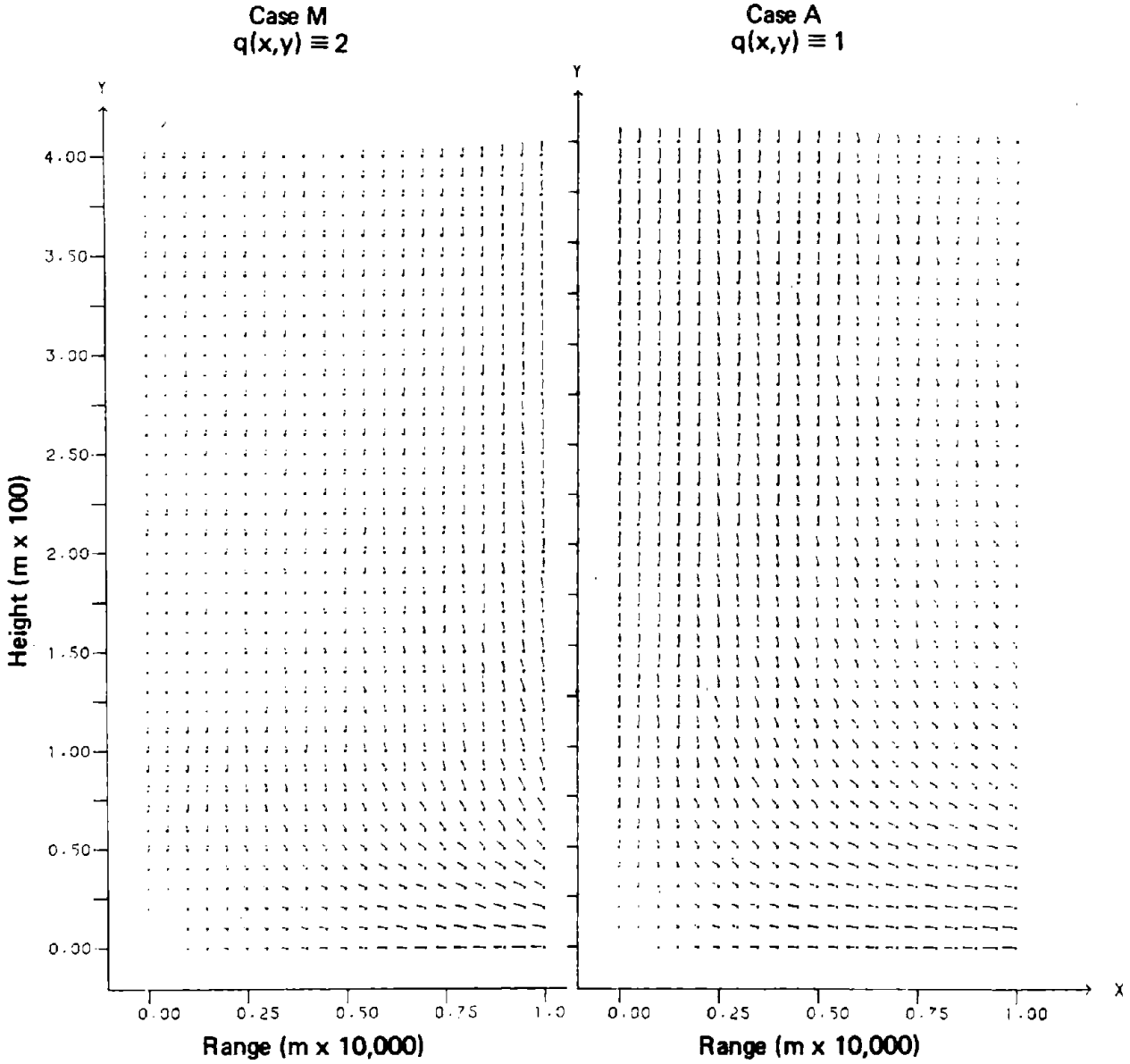


Fig. 24--Velocity fields generated for different spatially uniform heat release rates

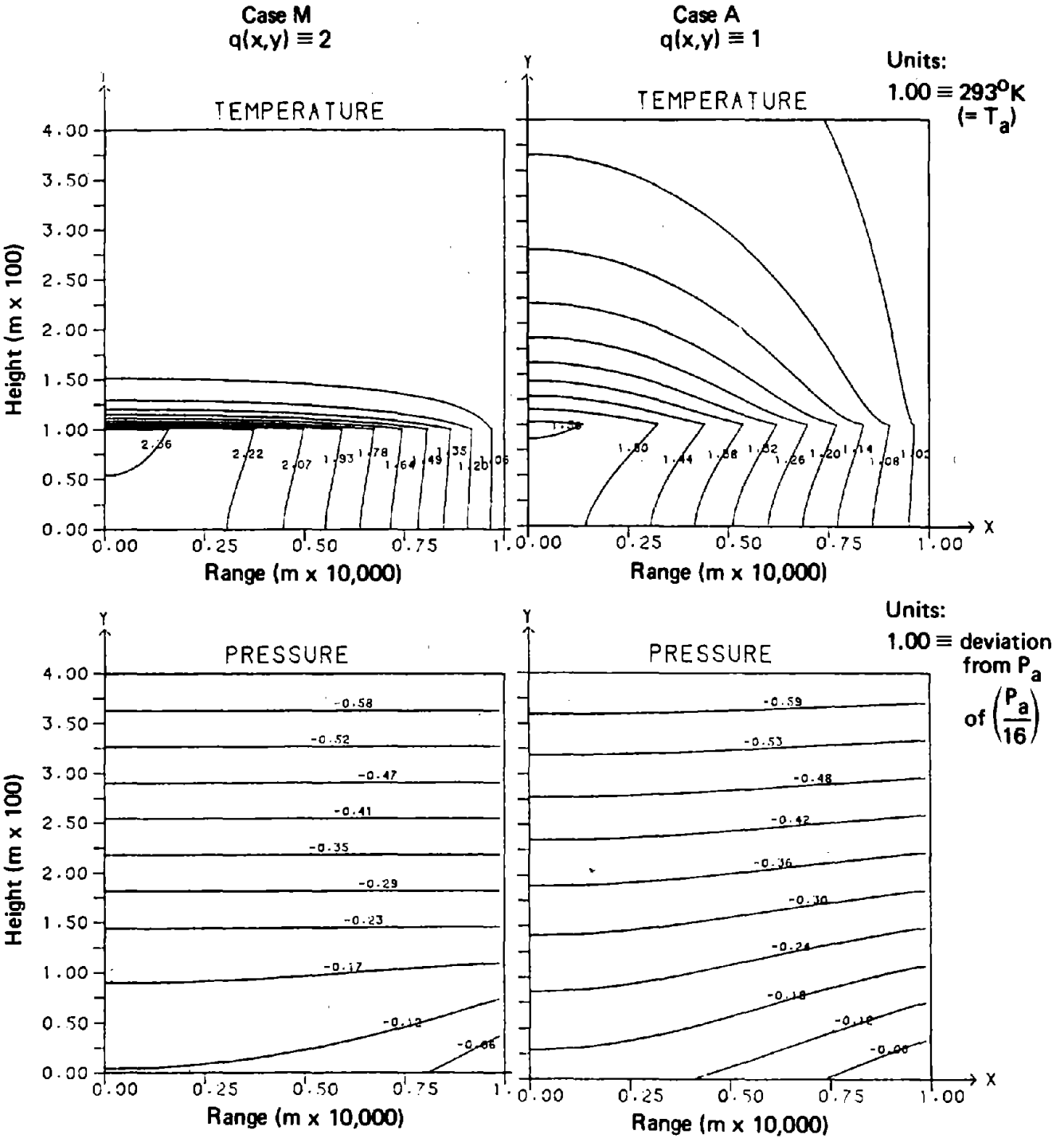


Fig. 25--Temperature and pressure contours generated for different spatially uniform heat release rates

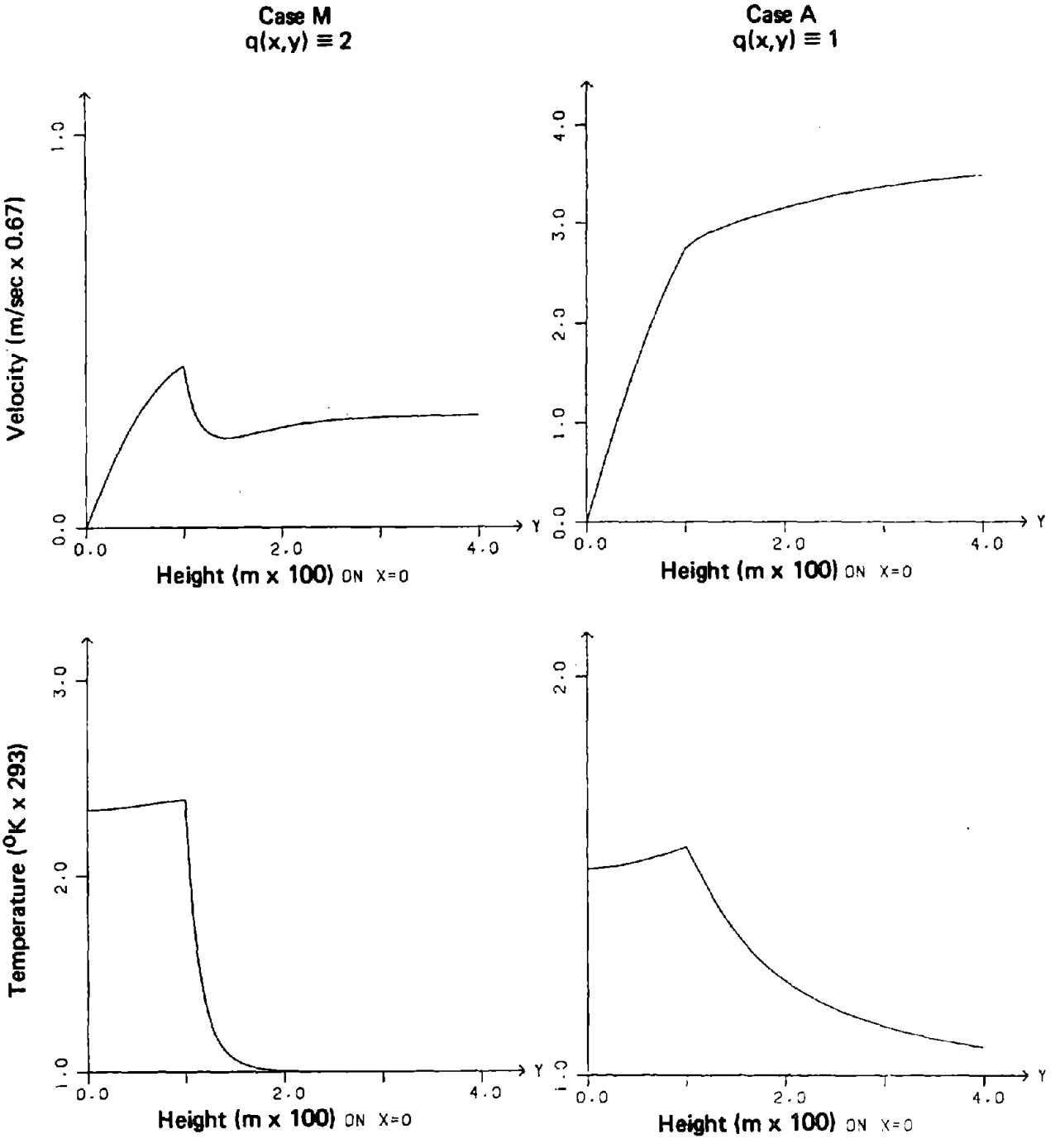


Fig. 26--Center-line velocities and temperatures generated for different spatially uniform heat release rates

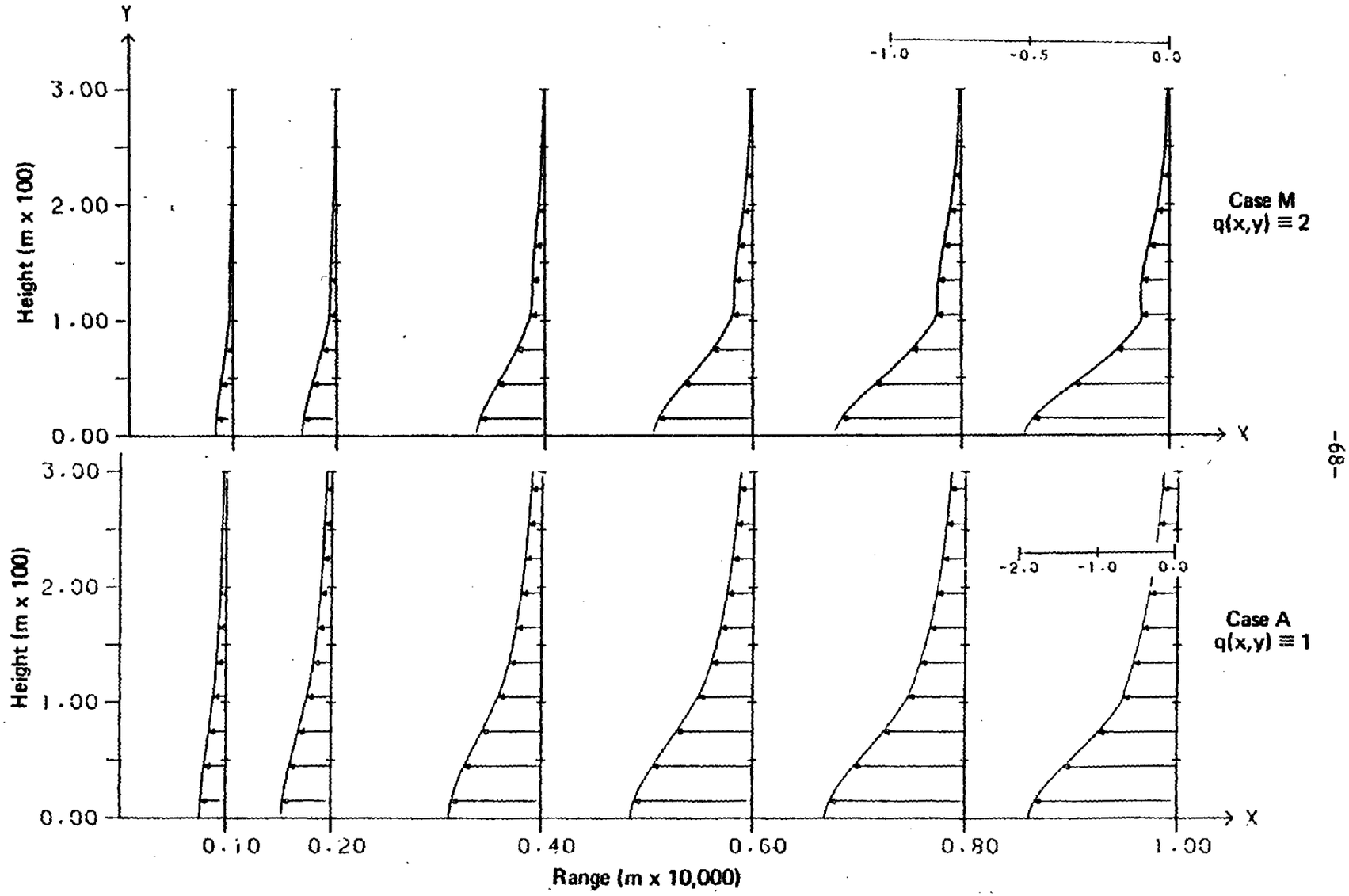


Fig. 27--Horizontal velocity profiles generated for different spatially uniform heat release rates

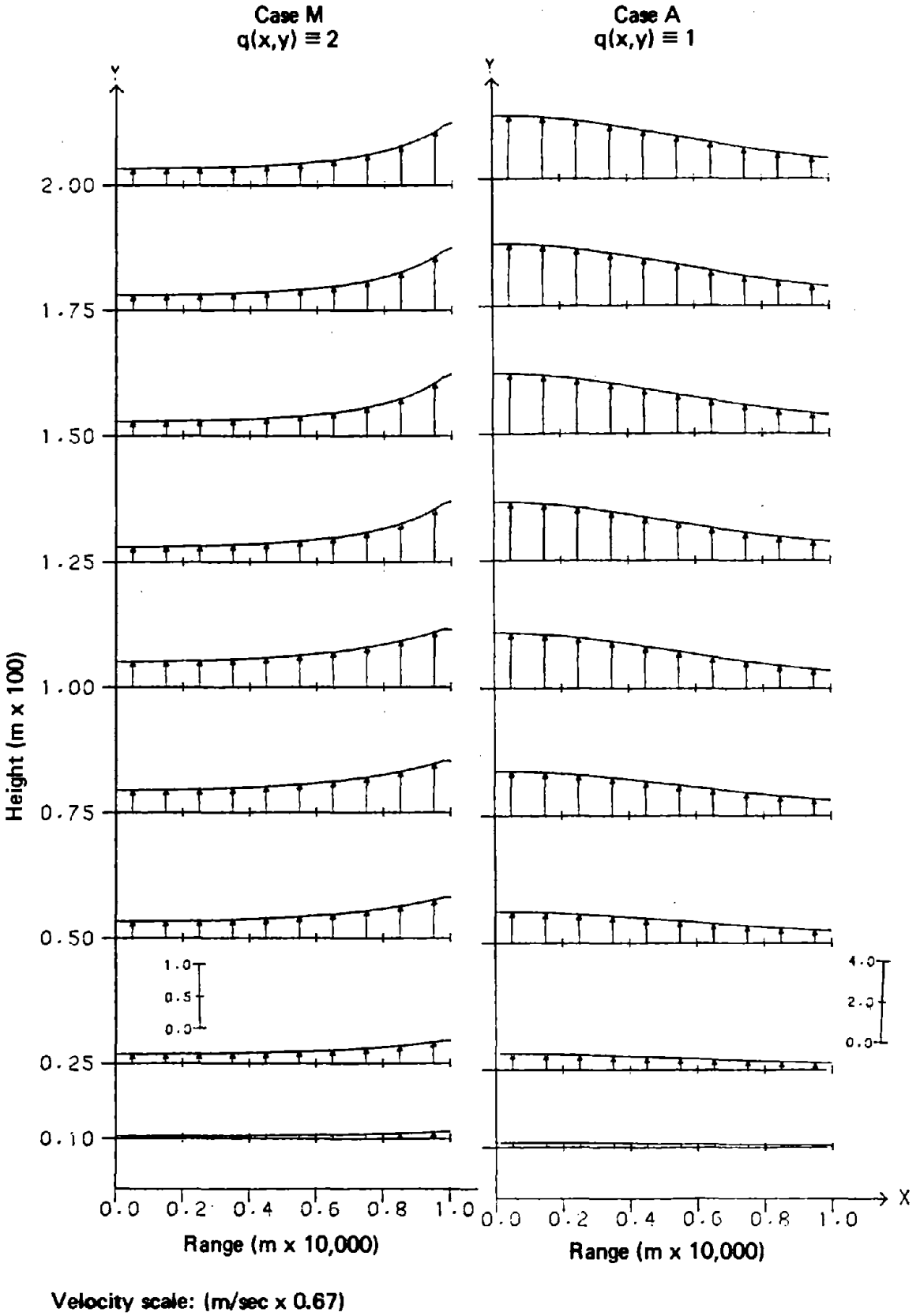


Fig. 28--Vertical velocity profiles generated for different spatially uniform heat release rates

or a decreased diffusion of heat (case L: Q = 1, K_1 = 0.2) engenders dynamically similar flows.

The vertical velocity profiles are fairly uniform for case H (Figs. 13 through 17), in which the coefficients for turbulent diffusion of heat and momentum were both increased three-fold above the baseline case. For cases L and M, the vertical velocities are greater at x = 1 than on the center line.

Further study is required to determine the most appropriate combinations of physically meaningful parameters such as diffusion coefficients, heating rates, and boundary velocities. However, these initial runs already suggest that a wider range of boundary velocities should be investigated.

A comparison of the ground-level and 600-m-altitude predictions for case M (Tables 8 and 9) with those for the baseline case A (Tables 2 and 3)shows that, as expected, the temperature at the ground rises significantly for the doubled-burning-rate case M. Comparison of the surface values (Tables 8 and 2) shows that the horizontal velocities are lower by a factor of more than 3 in the increased-burning-rate case--a result not immediately intuitive. With twice the heat addition rate (case M), the dominant effect is the diffusion of heat; in case A, however, the diffusion of momentum is greater than the diffusion of heat.

At 600 m (Tables 9 and 3), the horizontal velocities are negligible, and all vertical velocities except those at the outer boundary are a small fraction of the baseline velocities. The horizontal temperature and pressure gradients at 600 m are remarkably uniform.

Two calculations were made with periodic fires, i.e., with a sinusoidal variation in the burning rate such that the average was the same as in the baseline case. In case O, two burning rate peaks (cycles) were employed between the outer edge and the center line. In case P, a ten-peak fire was considered. Figures 29 through 33 compare the two-peak fire with the single uniform fire of case A. The flow fields (Fig. 29) show very little difference. Correspondingly, the pressure and temperature contours (Fig. 30) appear fairly similar. The centerline conditions (Fig. 31) are likewise quite similar. The horizontal

Table 8

GROUND-LEVEL NUMERICAL PREDICTIONS FOR PARAMETER CASE M
(INCREASED UNIFORM HEAT RELEASE RATE)

FIRE4 SOLUTION OF COMBUSTION LAYER BOUNDARY VALUE PROBLEM

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: M = 40 DX = 0.025 N = 240 DY = 0.025

ITERATION PARAMETERS: ETLINE = 0.001000 MAXITL = 20 ETOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 1.000 BETA = 1.500 BLOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.000	2.336	0.428	-0.114	0.000	0.000	0.000	1.338
2	0.025	-0.016	0.000	2.336	0.428	-0.114	0.001	0.006	-1.337	1.338
3	0.050	-0.032	0.000	2.335	0.428	-0.114	0.004	0.013	-1.339	1.340
4	0.075	-0.047	0.000	2.331	0.429	-0.114	0.007	0.020	-1.340	1.344
5	0.100	-0.063	0.000	2.326	0.430	-0.114	0.009	0.027	-1.344	1.350
6	0.125	-0.079	0.000	2.319	0.431	-0.114	0.012	0.034	-1.347	1.357
7	0.150	-0.095	0.000	2.311	0.433	-0.113	0.015	0.041	-1.352	1.367
8	0.175	-0.110	0.000	2.301	0.435	-0.113	0.018	0.048	-1.357	1.378
9	0.200	-0.126	0.000	2.289	0.437	-0.112	0.021	0.055	-1.364	1.391
10	0.225	-0.141	0.000	2.276	0.439	-0.112	0.024	0.062	-1.370	1.406
11	0.250	-0.157	0.000	2.261	0.442	-0.111	0.028	0.070	-1.378	1.422
12	0.275	-0.172	0.000	2.244	0.446	-0.110	0.031	0.078	-1.385	1.440
13	0.300	-0.187	0.000	2.225	0.449	-0.110	0.034	0.085	-1.394	1.459
14	0.325	-0.202	0.000	2.205	0.454	-0.109	0.038	0.093	-1.402	1.480
15	0.350	-0.217	0.000	2.183	0.458	-0.108	0.042	0.101	-1.408	1.501
16	0.375	-0.232	0.000	2.159	0.463	-0.107	0.046	0.109	-1.418	1.524
17	0.400	-0.247	0.000	2.133	0.469	-0.106	0.050	0.118	-1.424	1.547
18	0.425	-0.262	0.000	2.105	0.475	-0.104	0.054	0.127	-1.431	1.571
19	0.450	-0.276	0.000	2.076	0.482	-0.103	0.059	0.136	-1.435	1.596
20	0.475	-0.290	0.000	2.045	0.489	-0.101	0.064	0.145	-1.440	1.621
21	0.500	-0.304	0.000	2.012	0.497	-0.100	0.070	0.155	-1.441	1.646
22	0.525	-0.318	0.000	1.978	0.506	-0.098	0.076	0.165	-1.443	1.671
23	0.550	-0.332	0.000	1.941	0.515	-0.096	0.082	0.175	-1.441	1.696
24	0.575	-0.345	0.000	1.903	0.525	-0.094	0.089	0.186	-1.437	1.721
25	0.600	-0.359	0.000	1.863	0.537	-0.091	0.097	0.198	-1.431	1.746
26	0.625	-0.372	0.000	1.821	0.549	-0.089	0.105	0.210	-1.421	1.770
27	0.650	-0.384	0.000	1.778	0.563	-0.086	0.115	0.223	-1.408	1.793
28	0.675	-0.397	0.000	1.732	0.577	-0.083	0.125	0.238	-1.392	1.816
29	0.700	-0.409	0.000	1.685	0.593	-0.080	0.136	0.253	-1.372	1.838
30	0.725	-0.421	0.000	1.636	0.611	-0.076	0.149	0.269	-1.348	1.858
31	0.750	-0.432	0.000	1.586	0.631	-0.072	0.163	0.286	-1.317	1.878
32	0.775	-0.443	0.000	1.534	0.652	-0.068	0.179	0.306	-1.284	1.896
33	0.800	-0.454	0.000	1.480	0.676	-0.063	0.197	0.327	-1.244	1.913
34	0.825	-0.465	0.000	1.425	0.702	-0.058	0.218	0.350	-1.196	1.928
35	0.850	-0.474	0.000	1.368	0.731	-0.052	0.242	0.376	-1.145	1.942
36	0.875	-0.484	0.000	1.310	0.764	-0.046	0.269	0.405	-1.084	1.955
37	0.900	-0.493	0.000	1.250	0.800	-0.039	0.302	0.438	-1.018	1.967
38	0.925	-0.501	0.000	1.189	0.841	-0.031	0.340	0.476	-0.941	1.977
39	0.950	-0.509	0.000	1.127	0.887	-0.022	0.385	0.520	-0.856	1.986
40	0.975	-0.516	0.000	1.064	0.940	-0.012	0.440	0.573	-0.759	1.994
41	1.000	-0.523	0.000	1.000	1.000	0.000	0.471	0.573	-0.759	2.000

Table 9

600-m-ALTITUDE NUMERICAL PREDICTIONS FOR PARAMETER CASE M
(INCREASED UNIFORM HEAT RELEASE RATE)

FOR J = 241 (Y = 6.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.289	1.000	1.000	-0.961	0.000	0.000	0.000	0.000
2	0.025	0.000	0.289	1.000	1.000	-0.961	0.000	0.000	-0.001	0.000
3	0.050	0.000	0.289	1.000	1.000	-0.961	0.000	0.000	0.001	0.000
4	0.075	0.000	0.290	1.000	1.000	-0.961	0.000	0.000	-0.001	0.000
5	0.100	0.000	0.290	1.000	1.000	-0.961	0.000	0.000	0.001	0.000
6	0.125	0.000	0.291	1.000	1.000	-0.961	0.000	0.000	-0.001	0.000
7	0.150	0.000	0.292	1.000	1.000	-0.961	0.001	0.001	-0.001	0.000
8	0.175	0.000	0.294	1.000	1.000	-0.961	0.001	0.001	0.002	0.000
9	0.200	0.000	0.295	1.000	1.000	-0.961	0.001	0.001	-0.003	0.000
10	0.225	0.000	0.297	1.000	1.000	-0.961	0.001	0.001	0.005	0.000
11	0.250	0.000	0.299	1.000	1.000	-0.961	0.001	0.001	-0.004	0.000
12	0.275	0.000	0.302	1.000	1.000	-0.961	0.001	0.001	0.002	0.000
13	0.300	0.000	0.305	1.000	1.000	-0.961	0.001	0.001	0.001	0.000
14	0.325	0.000	0.308	1.000	1.000	-0.961	0.001	0.001	-0.001	0.000
15	0.350	0.000	0.312	1.000	1.000	-0.961	0.001	0.001	0.001	0.000
16	0.375	-0.001	0.316	1.000	1.000	-0.961	0.001	0.001	-0.001	0.000
17	0.400	-0.001	0.320	1.000	1.000	-0.960	0.001	0.001	-0.001	0.000
18	0.425	-0.001	0.326	1.000	1.000	-0.960	0.001	0.001	0.001	0.000
19	0.450	-0.001	0.331	1.000	1.000	-0.960	0.002	0.002	0.002	0.000
20	0.475	-0.001	0.338	1.000	1.000	-0.960	0.001	0.001	-0.002	0.000
21	0.500	-0.001	0.345	1.000	1.000	-0.960	0.001	0.001	0.001	0.000
22	0.525	-0.001	0.353	1.000	1.000	-0.960	0.002	0.002	0.000	0.000
23	0.550	-0.001	0.362	1.000	1.000	-0.960	0.001	0.001	-0.002	0.000
24	0.575	-0.001	0.372	1.000	1.000	-0.960	0.001	0.001	0.002	0.000
25	0.600	-0.001	0.384	1.000	1.000	-0.960	0.002	0.002	0.000	0.000
26	0.625	-0.001	0.396	1.000	1.000	-0.960	0.001	0.001	-0.001	0.000
27	0.650	-0.001	0.411	1.000	1.000	-0.960	0.001	0.001	0.000	0.000
28	0.675	-0.001	0.427	1.000	1.000	-0.960	0.001	0.001	-0.002	0.000
29	0.700	-0.001	0.445	1.000	1.000	-0.960	0.001	0.001	0.002	0.000
30	0.725	-0.001	0.465	1.000	1.000	-0.960	0.001	0.001	0.000	0.000
31	0.750	-0.001	0.488	1.000	1.000	-0.960	0.001	0.001	-0.001	0.000
32	0.775	-0.001	0.514	1.000	1.000	-0.960	0.001	0.001	0.000	0.000
33	0.800	-0.001	0.544	1.000	1.000	-0.960	0.001	0.001	0.000	0.000
34	0.825	-0.001	0.578	1.000	1.000	-0.960	0.001	0.001	0.000	0.000
35	0.850	-0.001	0.617	1.000	1.000	-0.960	0.000	0.001	-0.002	0.000
36	0.875	-0.001	0.662	1.000	1.000	-0.960	0.000	0.000	0.002	0.000
37	0.900	-0.001	0.715	1.000	1.000	-0.960	0.000	0.000	-0.001	0.000
38	0.925	-0.001	0.776	1.000	1.000	-0.960	-0.001	-0.001	0.000	0.000
39	0.950	-0.001	0.849	1.000	1.000	-0.960	-0.001	-0.001	0.000	0.000
40	0.975	-0.001	0.935	1.000	1.000	-0.960	-0.002	-0.002	0.000	0.000
41	1.000	-0.001	0.981	1.000	1.000	-0.960	-0.002	-0.002	0.000	0.000

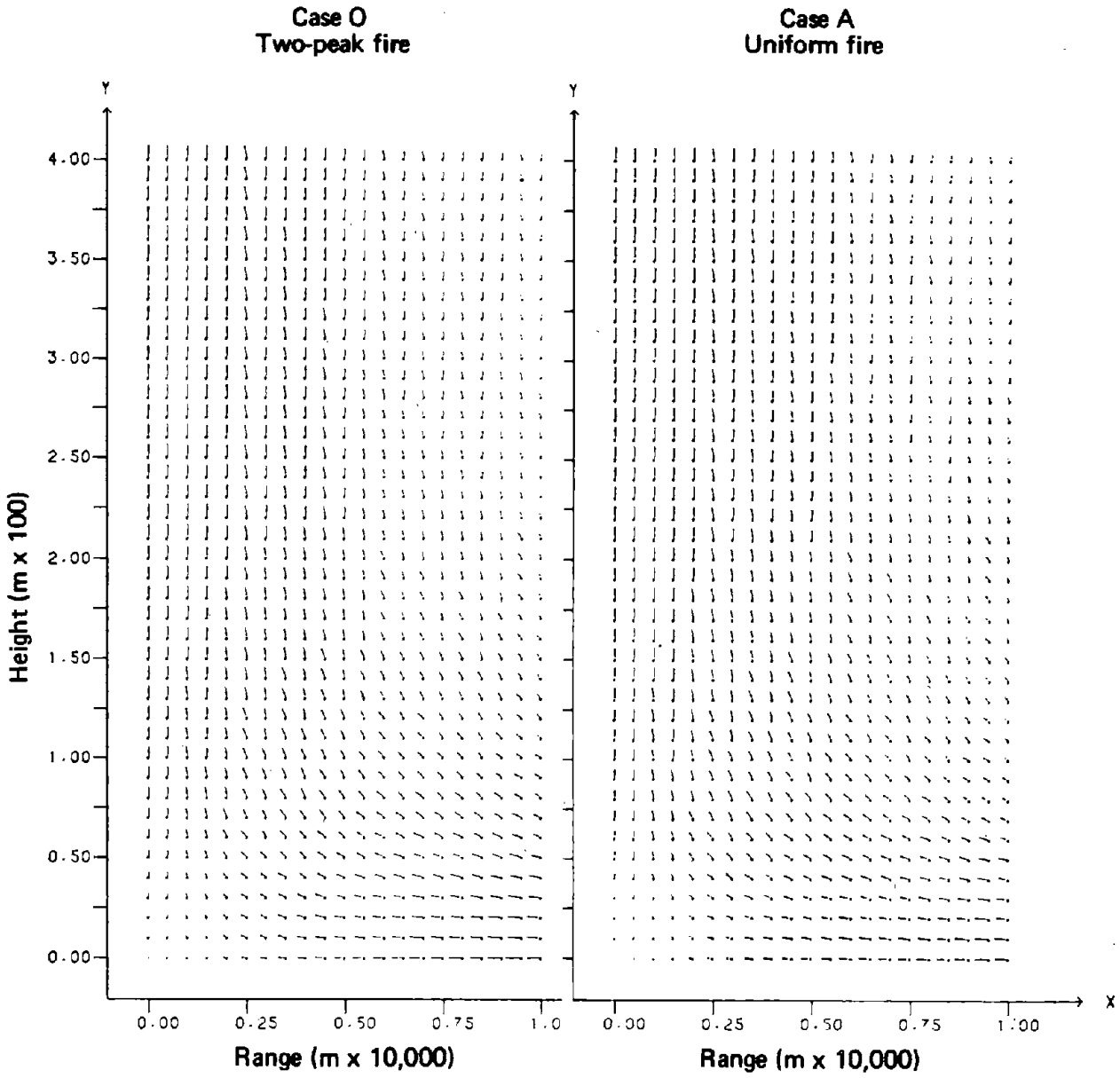


Fig. 29--Velocity fields generated for uniform and two-cycle oscillatory heat release rates

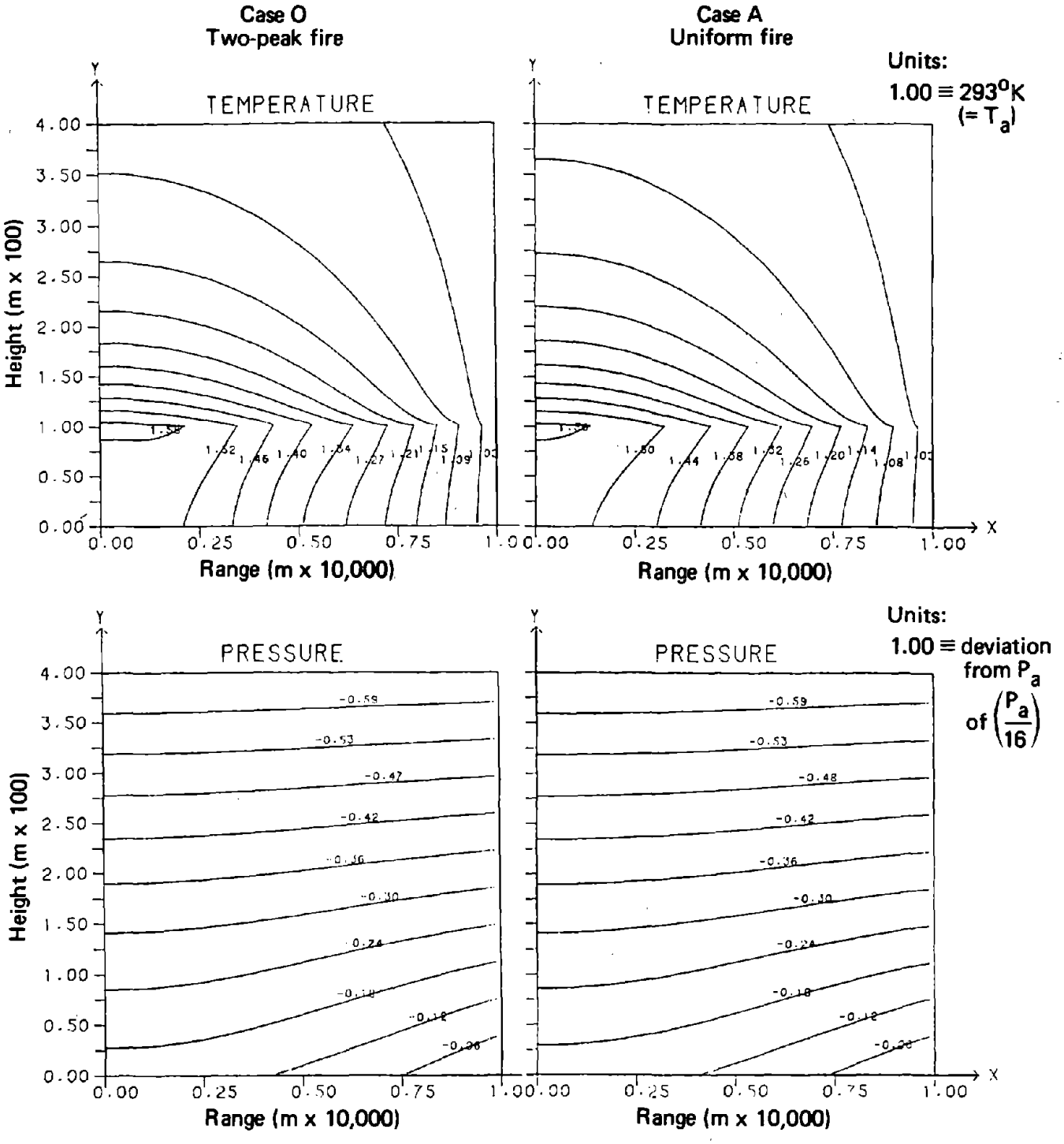


Fig. 30--Temperature and pressure contours generated for uniform and two-cycle oscillatory heat release rates

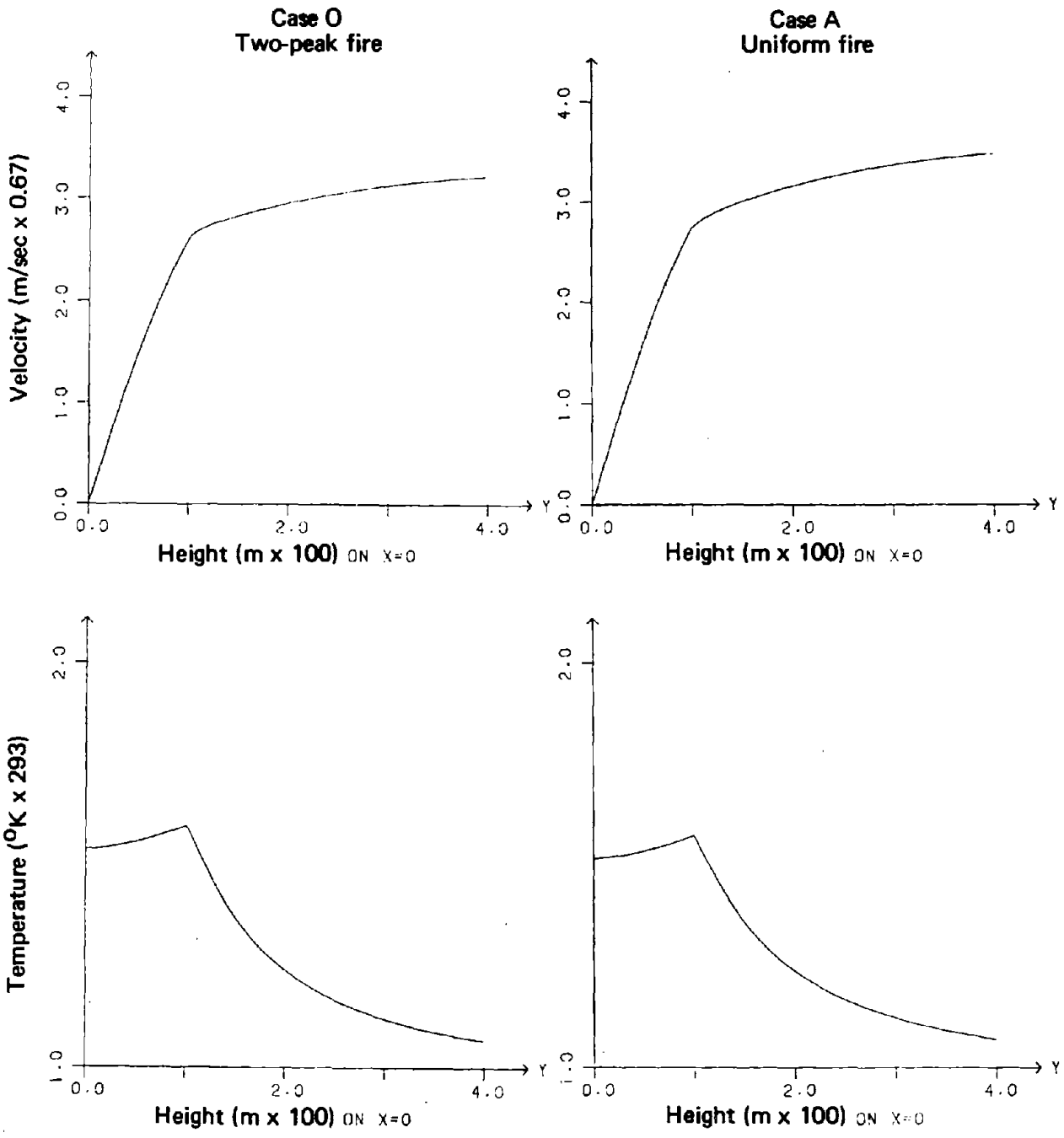
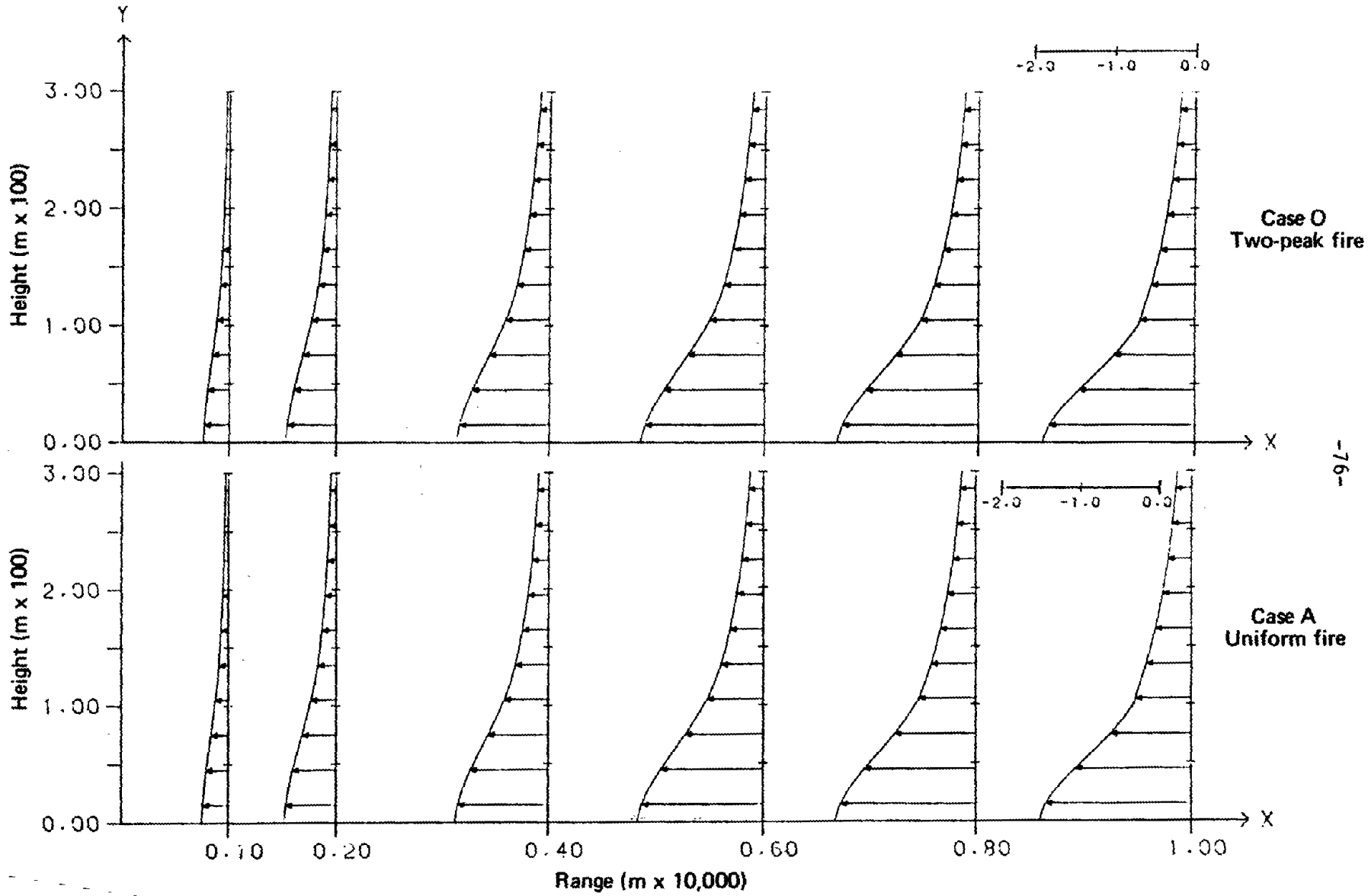


Fig. 31--Center-line velocities and temperatures generated for uniform and two-cycle oscillatory heat release rates



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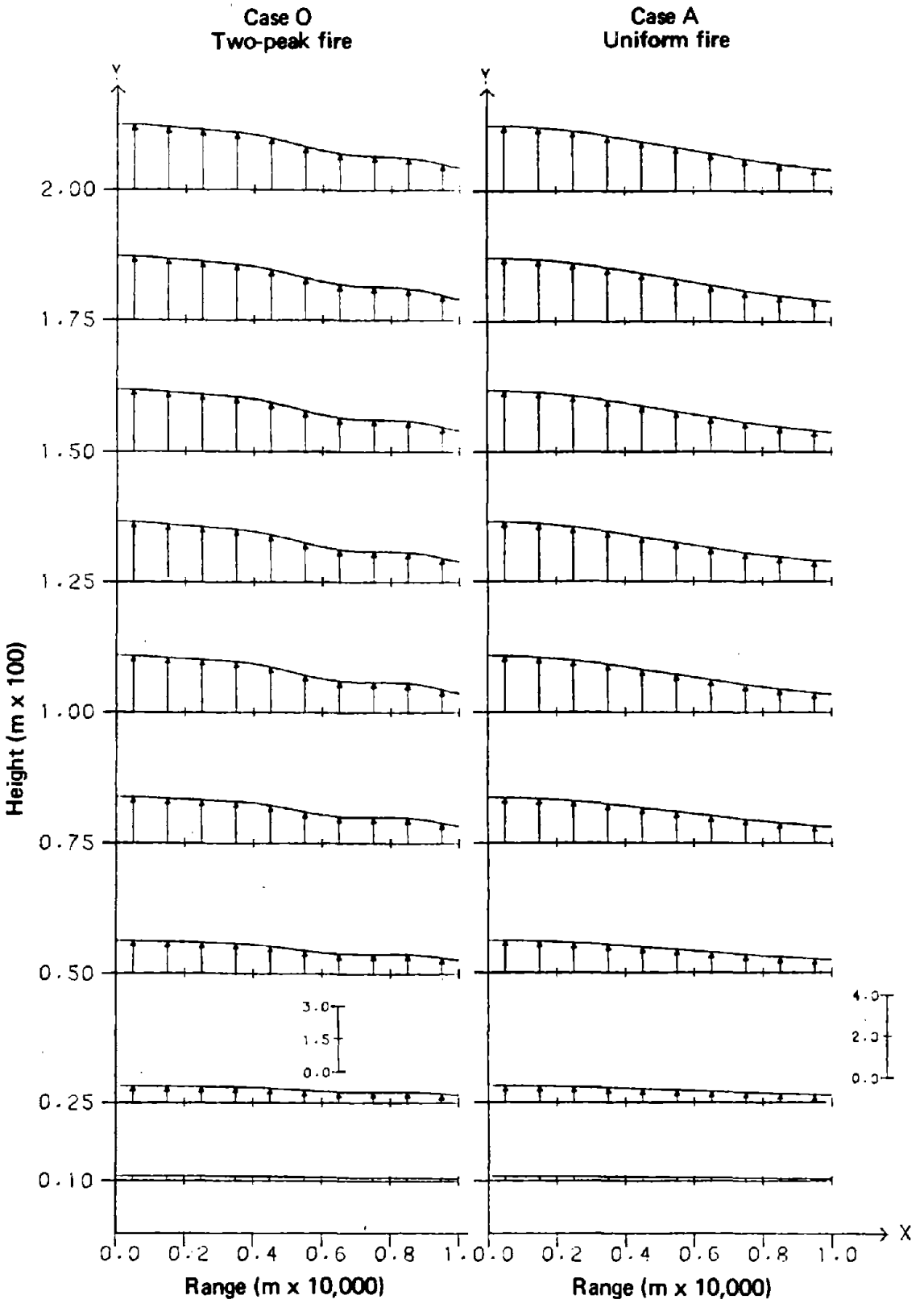


Fig. 33--Vertical velocity profiles generated for uniform and two-cycle oscillatory heat release rates

velocities (Fig. 32) show almost no difference. However, some difference in the vertical velocities (Fig. 33) does persist above the fire.

For the ten-peak fire of case P, very little difference from the baseline case is discernible in Figs. 34 through 38. Dispersive mechanisms are sufficient to make the individual fires unrecognizable beyond the burning region, or even in the burning region.

Case Q considers a fire with no combustion ($Q = 0$, $x < 1/3$, $y \leq 1$) in the inner third of the burning region. For comparison, the outer two-thirds of the combustion zone is assumed to burn with an increase such that the total amount of heat added is the same as in the baseline case. Figures 39 through 43 make the comparison. The velocity fields in Fig. 39 show relatively more motion up the plume for the "inner dead zone" fire. The differences are less obvious in the plots because of the differences in vector scaling. In fact, for the dead-zone fire, a much stronger central column velocity occurs, so that the inflow at the base and the rest of the column updraft appears to be smaller.

The temperature and pressure contours in Fig. 40 show that the highest temperatures occur at the inner edge of the burning region ($x = 1/3$) rather than at the center line--a result that is compatible with the competition between the heating source and the dissipative mechanisms. The center-line velocity and temperature contours in Fig. 41 show a much depressed temperature along the center line for the dead-zone case, but a much higher velocity. The dead zone appears to act as a flue or focusing chimney for the rising column.

Figure 42, which compares the horizontal velocities, shows some to be higher for case Q (again the cases are plotted with different scales). The vertical velocity profiles in Fig. 43 show a much larger velocity gradient near the center line for the dead-zone case (again, different scales). Both cases exhibit identical vertical velocities at $x = 1$--but the dead-zone case has a center-line velocity that, at the top of the plume, is 11 times greater than the boundary velocity, whereas the baseline case has a velocity ratio of approximately 3.5:1.

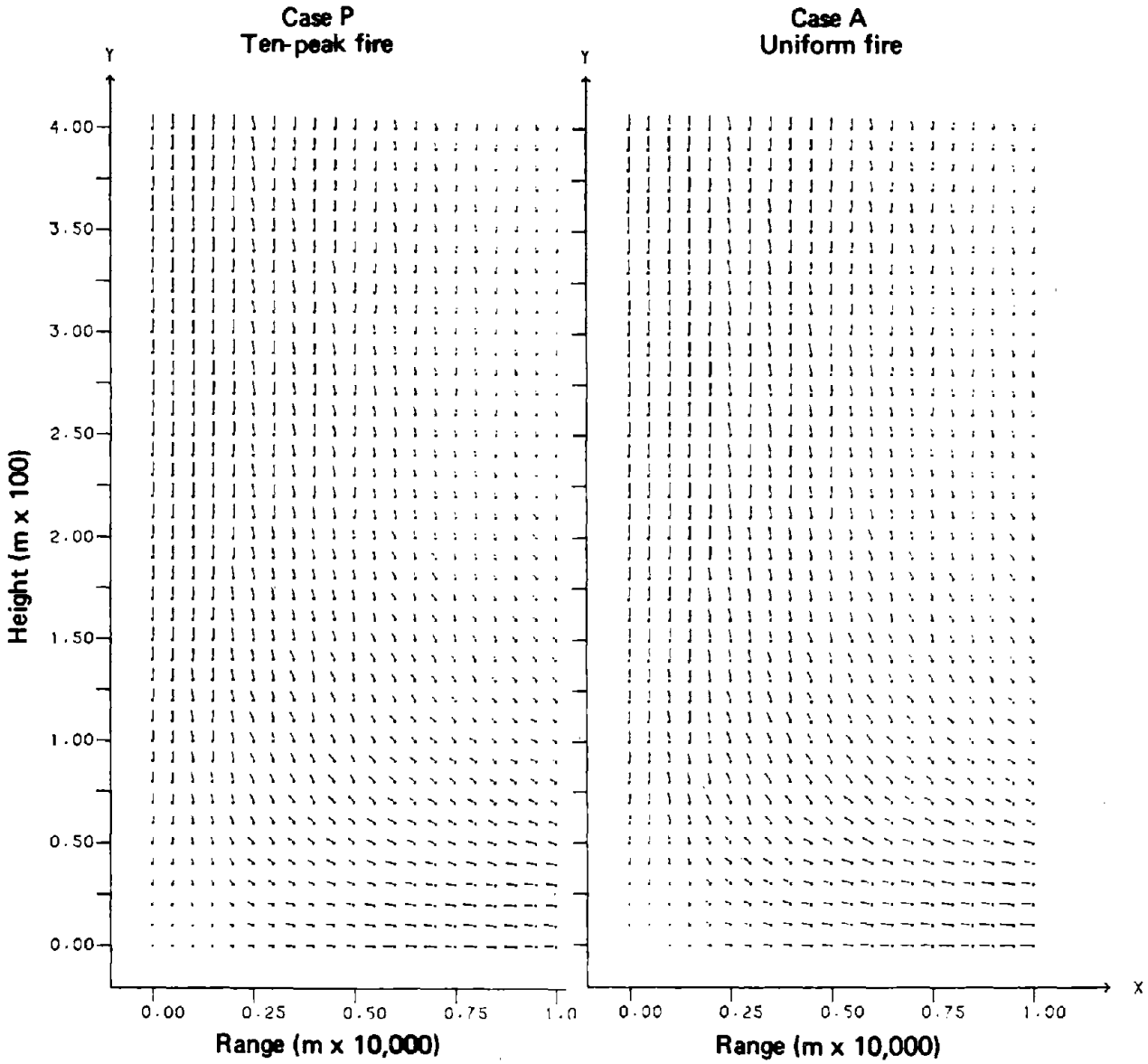


Fig. 34--Velocity fields generated for uniform and ten-cycle oscillatory heat release rates

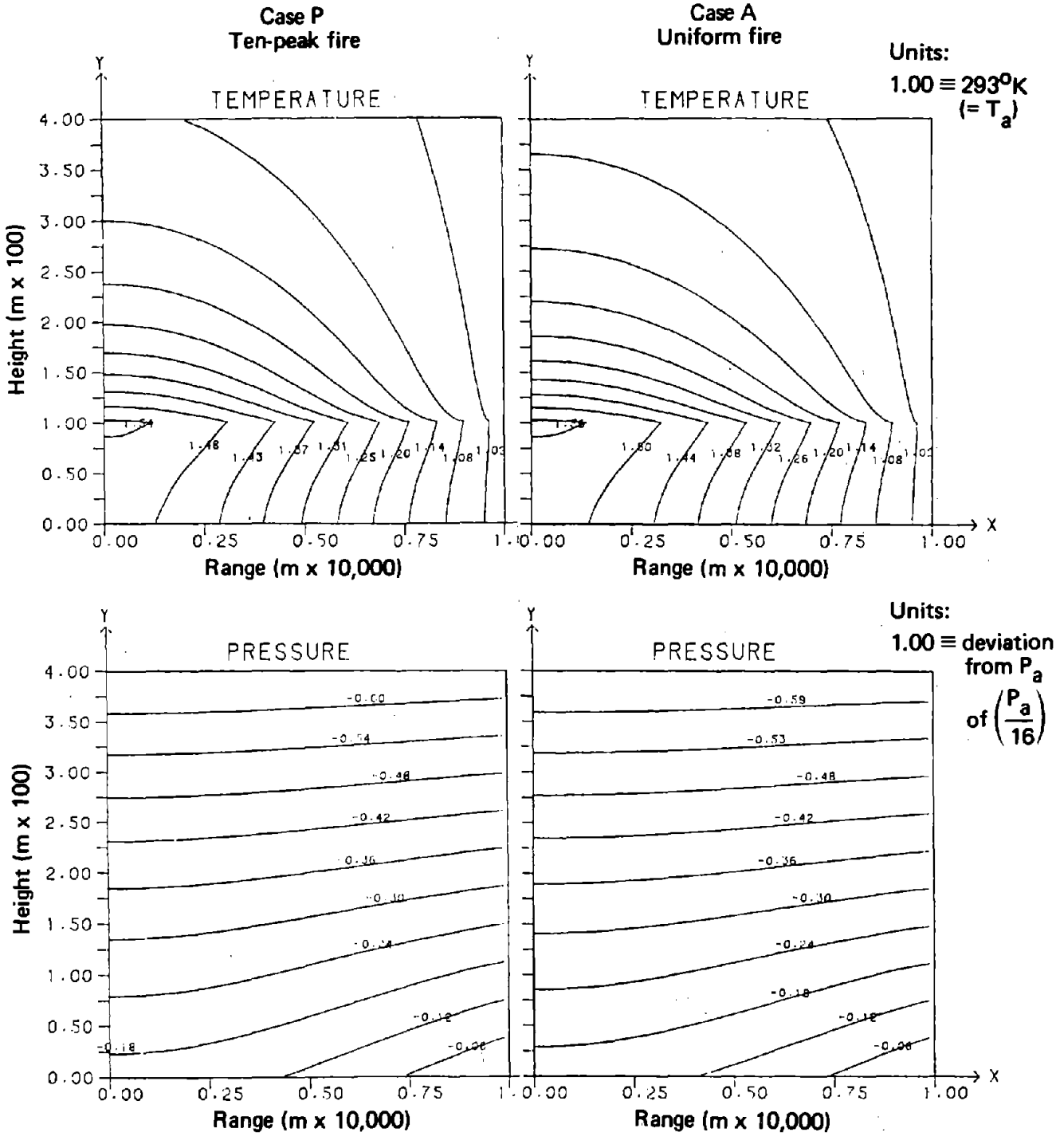


Fig. 35--Temperature and pressure contours generated for uniform and ten-cycle oscillatory heat release rates

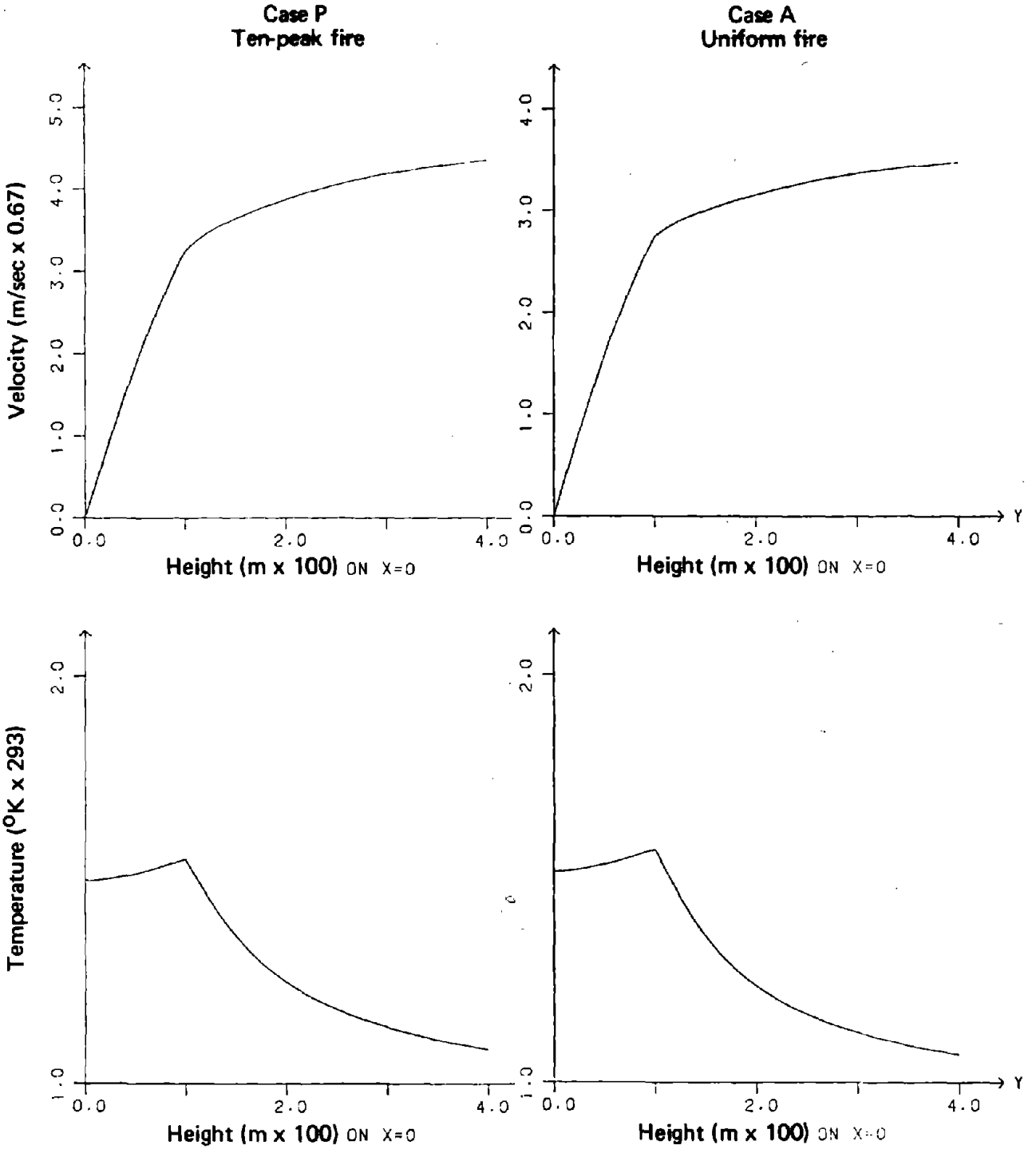
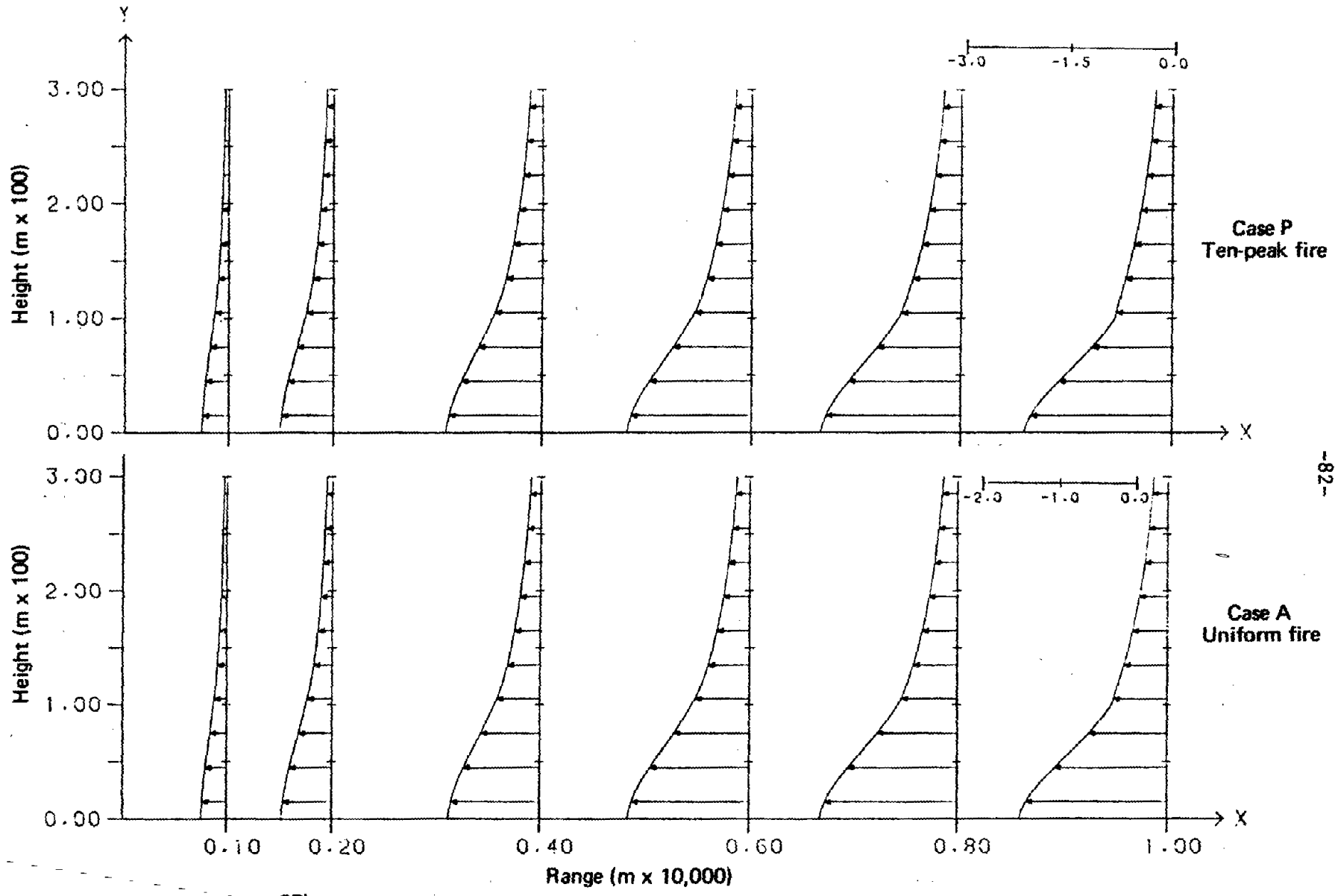


Fig. 36--Center-line velocities and temperatures generated for uniform and ten-cycle oscillatory heat release rates



...67)

...ted for uniform and ten-cycle oscillatory

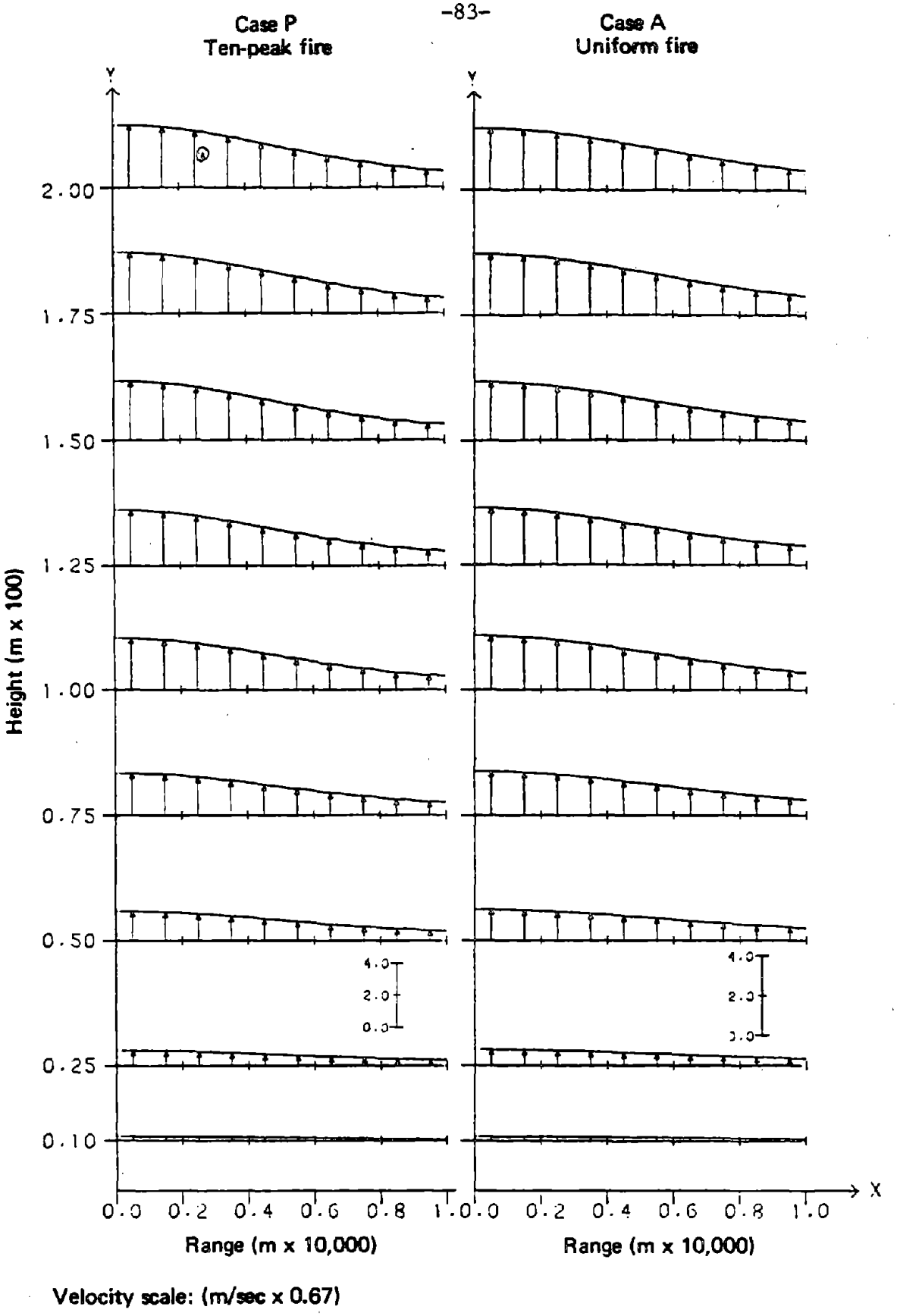


Fig. 38--Vertical velocity profiles generated for uniform and ten-cycle oscillatory heat release rates

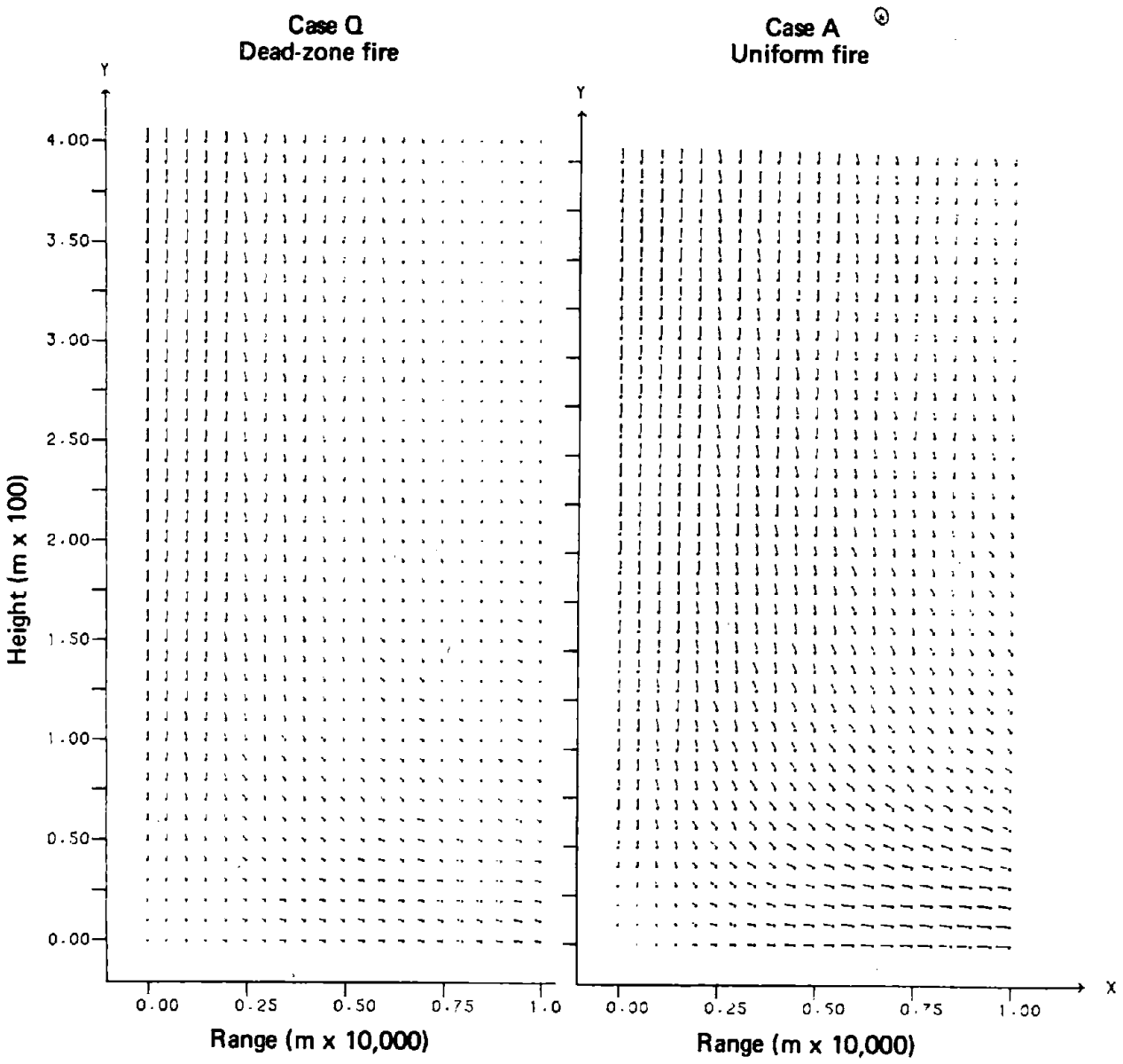


Fig. 39--Velocity fields generated for uniform and dead-zone heat release rates

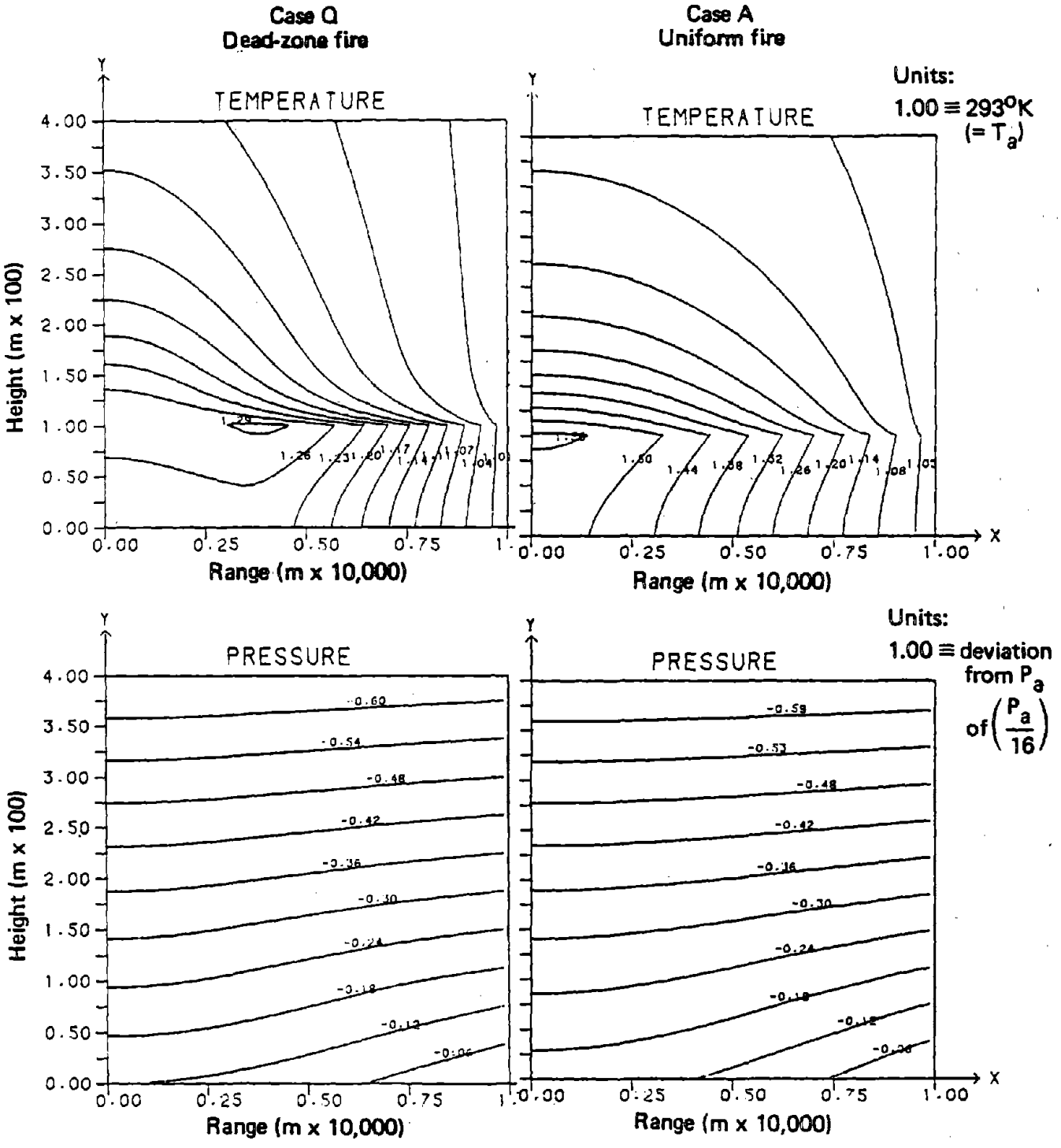


Fig. 40--Temperature and pressure contours generated for uniform and dead-zone heat release rates

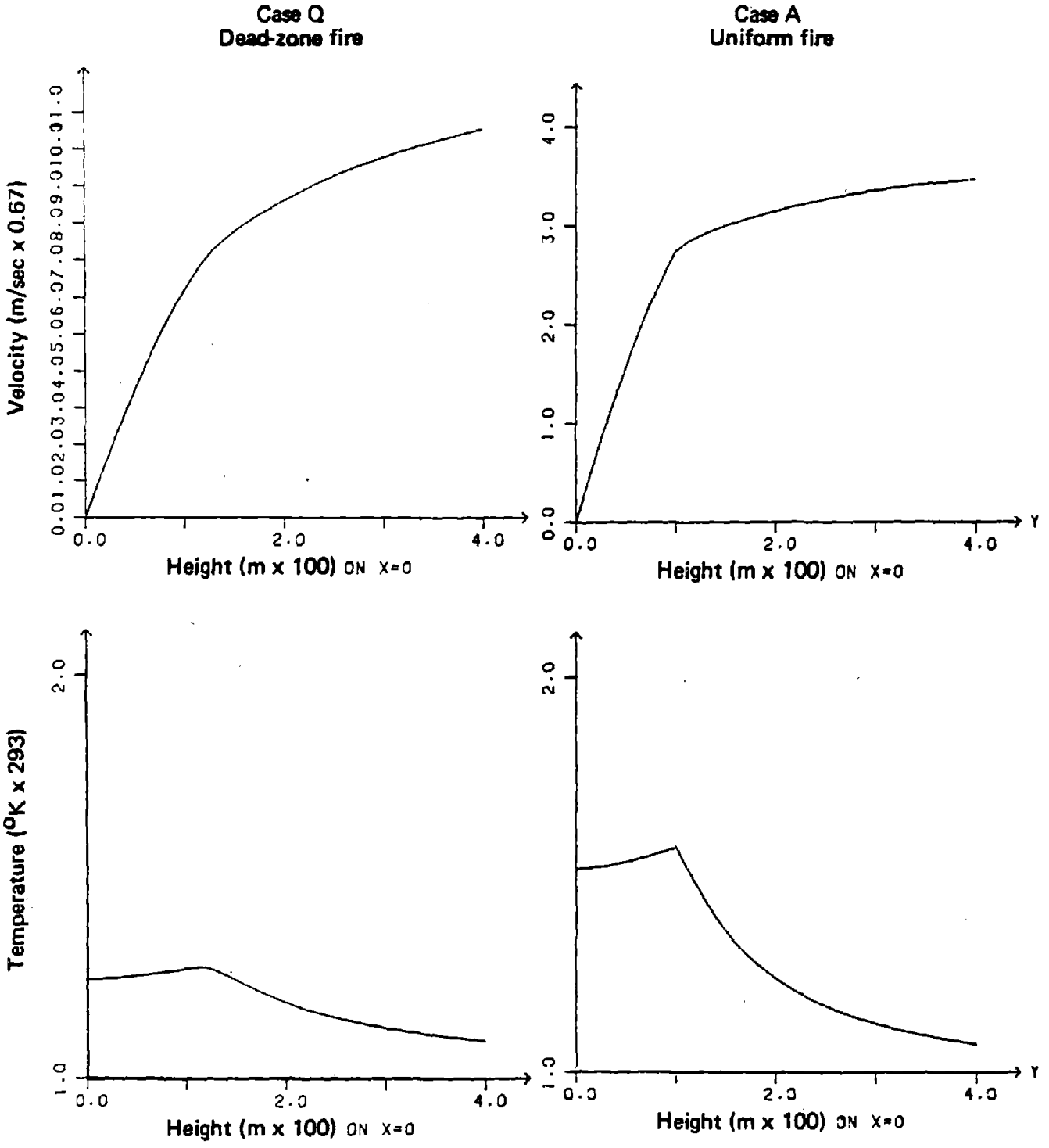
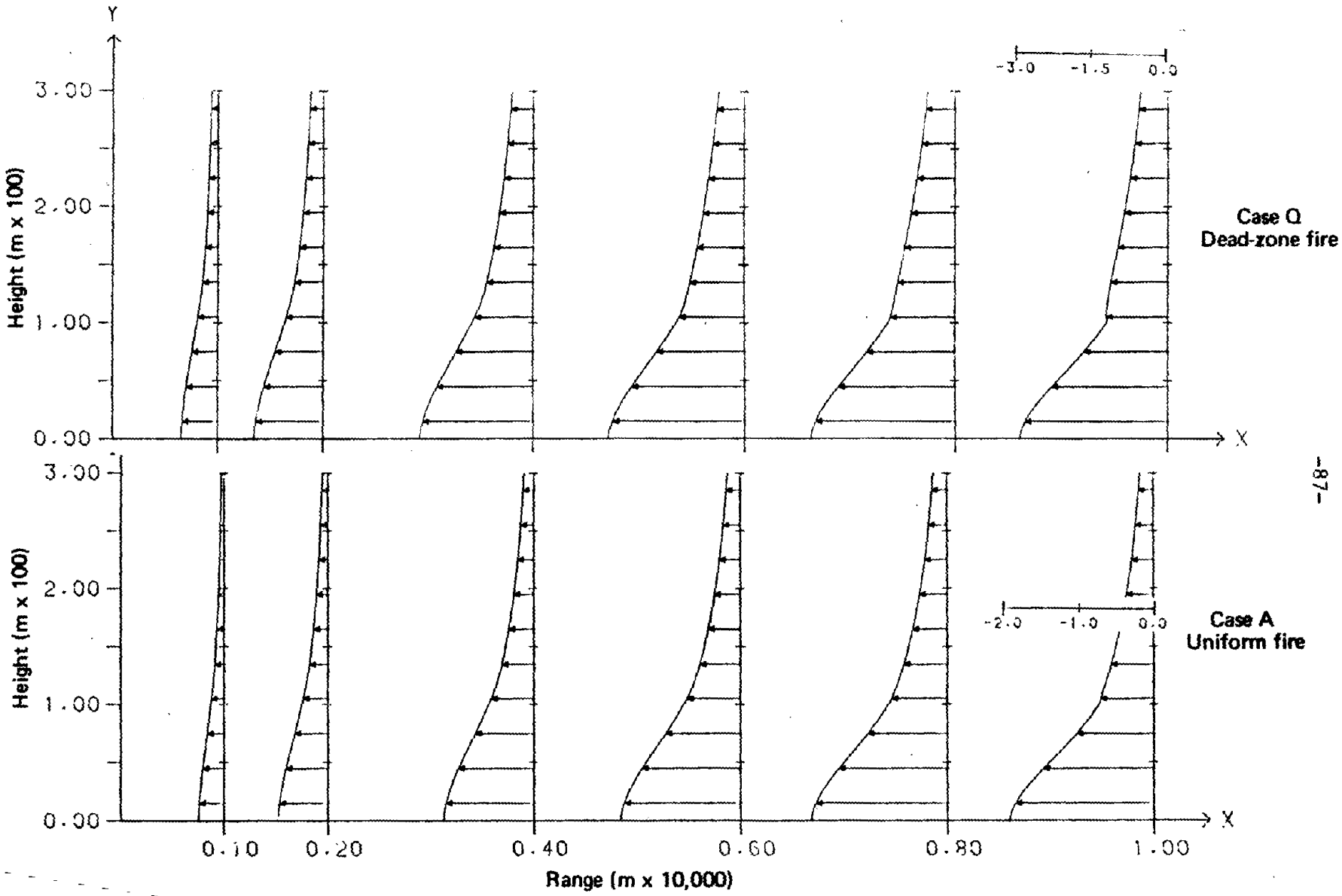


Fig. 41--Center-line velocities and temperatures generated for uniform and dead-zone heat release rates



for uniform and dead-zone heat release rates

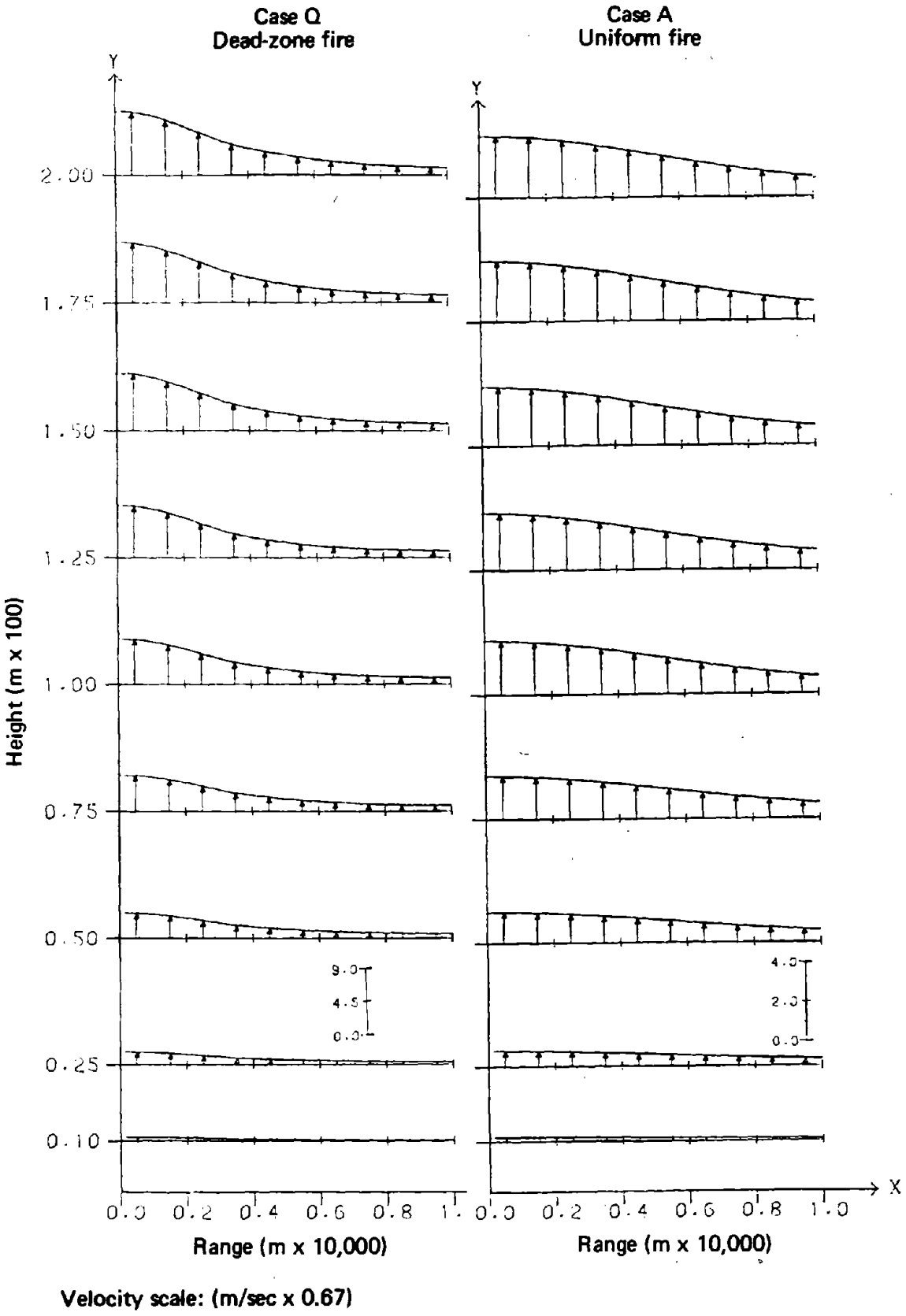


Fig. 43--Vertical velocity profiles generated for uniform and dead-zone heat release rates

Case Q is further elucidated in Tables 10 and 11. Ground-level velocities (Table 10) are appreciably higher than those in the baseline case (Table 2). The temperature at the center is lower, but the turbulent forces are more than three times higher than in the dead-zone fire case, with a peak just inside the inner edge of the fire. At 600 m (Table 11), horizontal velocities are much higher for case Q than for case A--nearly five times, in fact--though still small compared with the vertical velocities. Even here, turbulent stresses are nearly an order of magnitude higher than in the baseline case.

The Flambeau fire experiments were intended as models of large urban fires, with ricks of forest cuttings and brush separated by "streets." The tests resulted in fires approximately the size shown in Fig. 4 (p. 22). Application of our large-area-fire model to this configuration led to the results depicted in Figs. 44 through 48.

The velocity vectors of Fig. 44 are reassuringly similar to those of the baseline calculation (case A). The temperature and pressure contours of Fig. 45 also resemble those of the baseline calculation, but with somewhat different temperature profiles within the fire zone itself. The center-line temperature and velocity plots of Fig. 46 show the same general features as those of the baseline case. The horizontal velocity profiles in Fig. 47 show a persistence of the boundary velocities uniformly throughout the fire, with an exponential decay above $y = 1$. The vertical velocity profiles shown in Fig. 48 result in the plume formed over the fire having a fairly conventional velocity distribution, with a maximum along the center line and a decrease with distance toward $x = 1$. These results not only parallel those of many of the urban-fire parametric cases discussed above, but are satisfyingly close to actual observations of the Flambeau experiments. The similarity of the results to the observations/experiments is very encouraging.

Table 10

GROUND-LEVEL NUMERICAL PREDICTIONS FOR PARAMETER CASE Q (DEAD-ZONE FIRE)

FIRE: SOLUTION OF COMBUSTION LAYER BOUNDARY VALUE PROBLEM

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: M = 40 DX = 0.025 N = 240 DY = 0.025

ITERATION PARAMETERS: ETLINE = 0.001000 MAXITL = 20 ETOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 1.000 BETA = 1.500 BLOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.000	1.246	0.802	-0.122	0.000	0.000	0.000	-0.032
2	0.025	-0.187	0.000	1.246	0.802	-0.122	0.005	1.126	0.032	-0.032
3	0.050	-0.373	0.000	1.246	0.802	-0.122	0.014	2.222	0.032	-0.032
4	0.075	-0.556	0.000	1.246	0.802	-0.121	0.023	3.254	0.031	-0.032
5	0.100	-0.735	0.000	1.246	0.802	-0.121	0.031	4.192	0.028	-0.033
6	0.125	-0.909	0.000	1.247	0.802	-0.120	0.040	5.016	0.028	-0.033
7	0.150	-1.076	0.000	1.247	0.802	-0.119	0.048	5.707	0.025	-0.033
8	0.175	-1.237	0.000	1.247	0.802	-0.117	0.055	6.261	0.024	-0.033
9	0.200	-1.389	0.000	1.247	0.802	-0.116	0.063	6.670	0.021	-0.033
10	0.225	-1.533	0.000	1.248	0.802	-0.114	0.069	6.945	0.018	-0.033
11	0.250	-1.669	0.000	1.248	0.801	-0.112	0.075	7.086	0.017	-0.033
12	0.275	-1.795	0.000	1.248	0.801	-0.110	0.081	7.112	0.014	-0.033
13	0.300	-1.913	0.000	1.248	0.801	-0.108	0.086	7.034	0.011	-0.033
14	0.325	-2.022	0.000	1.249	0.801	-0.106	0.090	6.870	0.010	-0.033
15	0.350	-2.123	0.000	1.249	0.801	-0.104	0.097	6.637	-1.268	1.297
16	0.375	-2.215	0.000	1.248	0.801	-0.101	0.107	6.359	-1.159	1.297
17	0.400	-2.299	0.000	1.245	0.803	-0.099	0.115	6.048	-1.053	1.298
18	0.425	-2.375	0.000	1.241	0.806	-0.096	0.123	5.710	-0.947	1.298
19	0.450	-2.445	0.000	1.236	0.809	-0.092	0.130	5.357	-0.849	1.299
20	0.475	-2.507	0.000	1.230	0.813	-0.089	0.136	4.997	-0.754	1.300
21	0.500	-2.564	0.000	1.223	0.818	-0.086	0.142	4.636	-0.665	1.302
22	0.525	-2.615	0.000	1.215	0.823	-0.082	0.147	4.280	-0.582	1.303
23	0.550	-2.660	0.000	1.206	0.829	-0.078	0.151	3.934	-0.506	1.304
24	0.575	-2.700	0.000	1.197	0.836	-0.074	0.156	3.600	-0.436	1.306
25	0.600	-2.736	0.000	1.187	0.843	-0.070	0.159	3.284	-0.372	1.307
26	0.625	-2.768	0.000	1.176	0.850	-0.066	0.162	2.980	-0.314	1.309
27	0.650	-2.796	0.000	1.166	0.858	-0.062	0.165	2.699	-0.262	1.311
28	0.675	-2.821	0.000	1.155	0.866	-0.058	0.168	2.435	-0.214	1.312
29	0.700	-2.843	0.000	1.143	0.875	-0.054	0.170	2.189	-0.173	1.314
30	0.725	-2.861	0.000	1.132	0.884	-0.050	0.172	1.964	-0.136	1.315
31	0.750	-2.878	0.000	1.120	0.893	-0.045	0.174	1.755	-0.102	1.317
32	0.775	-2.892	0.000	1.108	0.903	-0.041	0.176	1.566	-0.073	1.318
33	0.800	-2.905	0.000	1.096	0.912	-0.037	0.177	1.392	-0.048	1.320
34	0.825	-2.915	0.000	1.084	0.923	-0.032	0.179	1.235	-0.026	1.321
35	0.850	-2.924	0.000	1.072	0.933	-0.028	0.180	1.093	-0.006	1.323
36	0.875	-2.932	0.000	1.060	0.944	-0.023	0.181	0.964	0.011	1.324
37	0.900	-2.938	0.000	1.048	0.954	-0.019	0.183	0.849	0.025	1.325
38	0.925	-2.944	0.000	1.036	0.965	-0.014	0.184	0.746	0.038	1.327
39	0.950	-2.948	0.000	1.024	0.977	-0.009	0.185	0.655	0.048	1.328
40	0.975	-2.952	0.000	1.012	0.988	-0.005	0.186	0.571	0.057	1.329

Table 11

600-m-ALTITUDE NUMERICAL PREDICTIONS FOR PARAMETER CASE Q
(DEAD-ZONE FIRE)

FOR J = 241 (Y = 6.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	11.315	1.057	0.946	-0.960	0.000	0.000	0.000	-0.006
2	0.025	-0.009	11.273	1.057	0.946	-0.960	0.000	0.045	-0.125	-0.006
3	0.050	-0.018	11.149	1.057	0.946	-0.960	0.000	0.088	-0.123	-0.006
4	0.075	-0.027	10.947	1.056	0.947	-0.960	0.000	0.129	-0.120	-0.006
5	0.100	-0.035	10.674	1.056	0.947	-0.960	0.000	0.167	-0.115	-0.006
6	0.125	-0.044	10.337	1.055	0.948	-0.960	0.000	0.201	-0.109	-0.006
7	0.150	-0.052	9.945	1.055	0.948	-0.960	0.000	0.230	-0.103	-0.005
8	0.175	-0.060	9.510	1.054	0.949	-0.960	0.000	0.253	-0.096	-0.005
9	0.200	-0.067	9.042	1.053	0.950	-0.960	0.000	0.272	-0.087	-0.005
10	0.225	-0.075	8.552	1.052	0.951	-0.960	0.000	0.285	-0.082	-0.005
11	0.250	-0.081	8.048	1.050	0.952	-0.960	0.000	0.294	-0.071	-0.005
12	0.275	-0.088	7.541	1.049	0.953	-0.960	0.000	0.298	-0.066	-0.005
13	0.300	-0.094	7.037	1.048	0.954	-0.960	0.000	0.298	-0.056	-0.005
14	0.325	-0.100	6.544	1.046	0.956	-0.960	0.000	0.295	-0.051	-0.005
15	0.350	-0.105	6.101	1.045	0.957	-0.960	0.000	0.291	-0.042	-0.004
16	0.375	-0.110	5.713	1.043	0.959	-0.960	0.000	0.285	-0.037	-0.004
17	0.400	-0.115	5.343	1.042	0.960	-0.960	0.000	0.279	-0.032	-0.004
18	0.425	-0.119	4.991	1.040	0.962	-0.960	0.000	0.271	-0.027	-0.004
19	0.450	-0.123	4.658	1.038	0.963	-0.960	0.000	0.261	-0.021	-0.004
20	0.475	-0.127	4.343	1.037	0.965	-0.960	0.000	0.251	-0.020	-0.004
21	0.500	-0.130	4.047	1.035	0.966	-0.960	0.000	0.240	-0.014	-0.003
22	0.525	-0.133	3.768	1.033	0.968	-0.960	0.000	0.229	-0.011	-0.003
23	0.550	-0.136	3.506	1.031	0.970	-0.960	0.000	0.218	-0.010	-0.003
24	0.575	-0.139	3.260	1.030	0.971	-0.960	0.000	0.206	-0.005	-0.003
25	0.600	-0.141	3.031	1.028	0.973	-0.960	0.000	0.195	-0.002	-0.003
26	0.625	-0.143	2.816	1.026	0.975	-0.960	0.000	0.184	-0.002	-0.002
27	0.650	-0.144	2.616	1.024	0.976	-0.960	0.000	0.173	0.001	-0.002
28	0.675	-0.146	2.429	1.022	0.978	-0.960	0.001	0.162	0.004	-0.002
29	0.700	-0.147	2.256	1.021	0.980	-0.960	0.001	0.152	0.002	-0.002
30	0.725	-0.148	2.095	1.019	0.981	-0.960	0.001	0.142	0.006	-0.002
31	0.750	-0.149	1.945	1.017	0.983	-0.960	0.001	0.132	0.006	-0.002
32	0.775	-0.150	1.807	1.015	0.985	-0.960	0.001	0.123	0.006	-0.001
33	0.800	-0.151	1.679	1.014	0.987	-0.960	0.001	0.115	0.008	-0.001
34	0.825	-0.151	1.560	1.012	0.988	-0.960	0.001	0.107	0.008	-0.001
35	0.850	-0.152	1.450	1.010	0.990	-0.960	0.001	0.099	0.011	-0.001
36	0.875	-0.152	1.349	1.008	0.992	-0.960	0.001	0.092	0.006	-0.001
37	0.900	-0.152	1.255	1.007	0.993	-0.960	0.001	0.086	0.013	-0.001
38	0.925	-0.152	1.169	1.005	0.995	-0.960	0.001	0.079	0.006	0.000
39	0.950	-0.152	1.090	1.003	0.997	-0.960	0.001	0.074	0.012	0.000
40	0.975	-0.152	1.017	1.002	0.998	-0.960	0.001	0.067	0.010	0.000
41	1.000	-0.152	0.981	1.000	1.000	-0.960	0.002	0.067	0.010	0.000

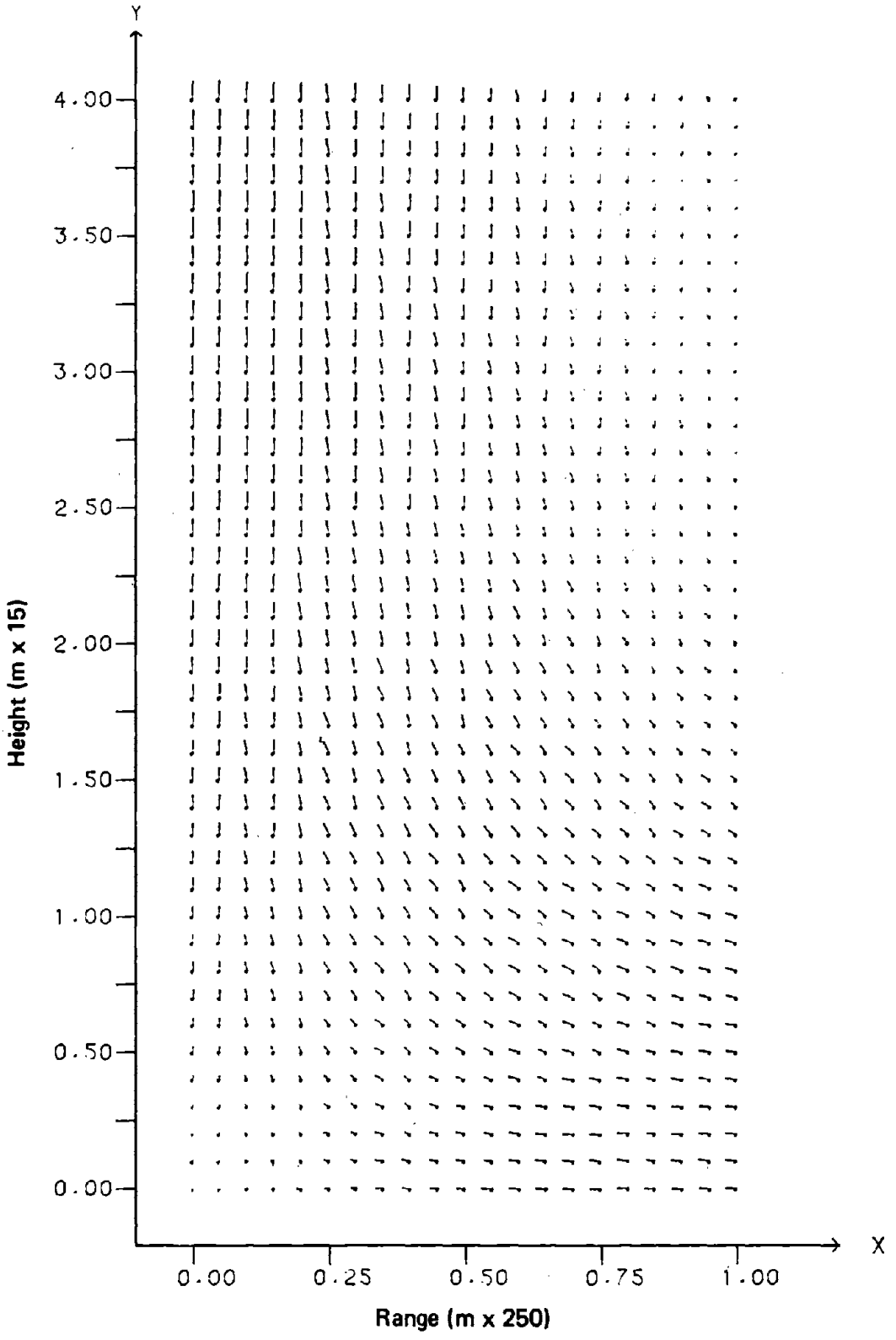


Fig. 44--Velocity field predicted for typical multiple-fuel-bed Flambeau fire (cf. Fig. 4)

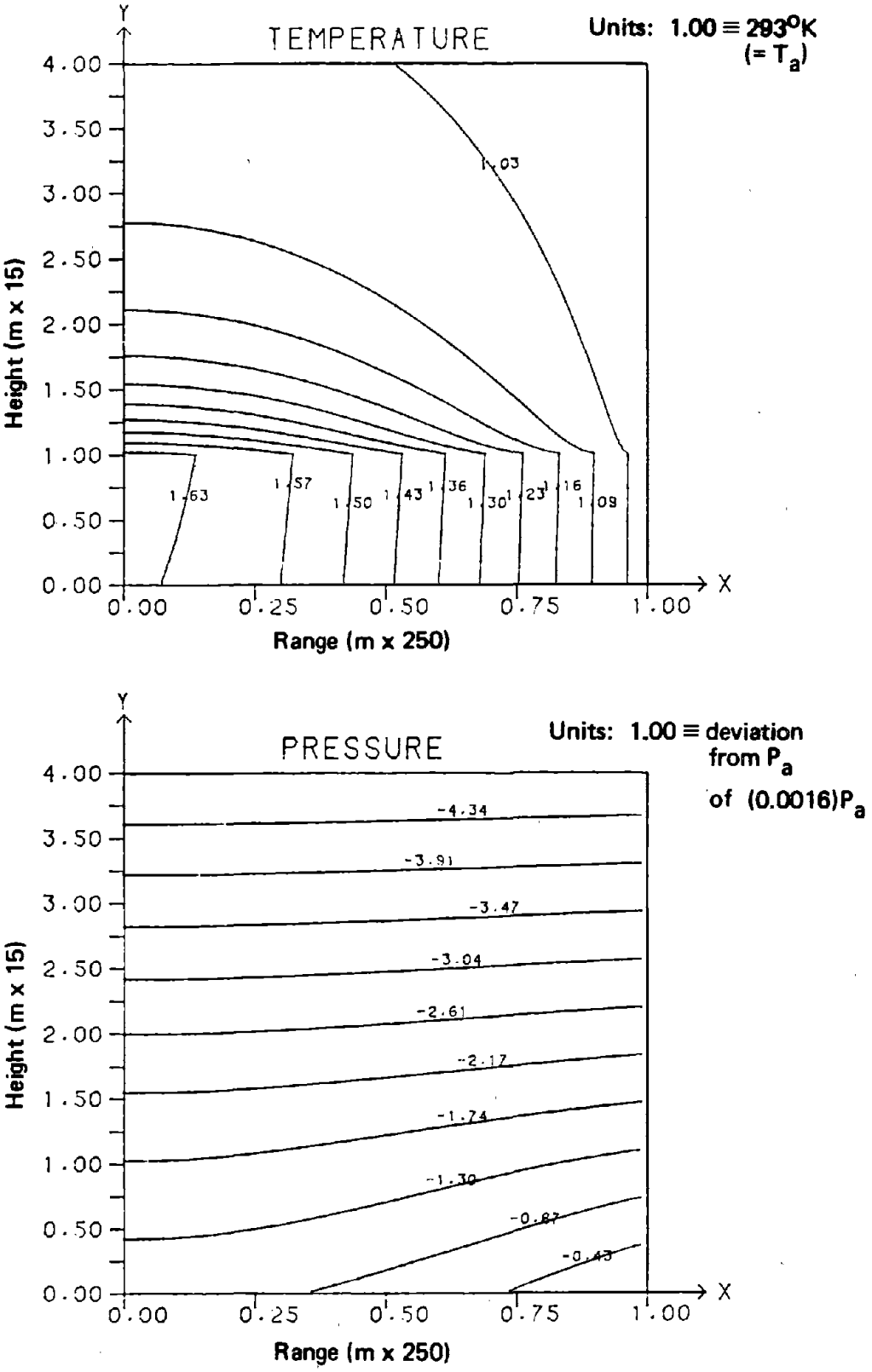


Fig. 45--Temperature and pressure contours predicted for typical multiple-fuel-bed Flambeau fire

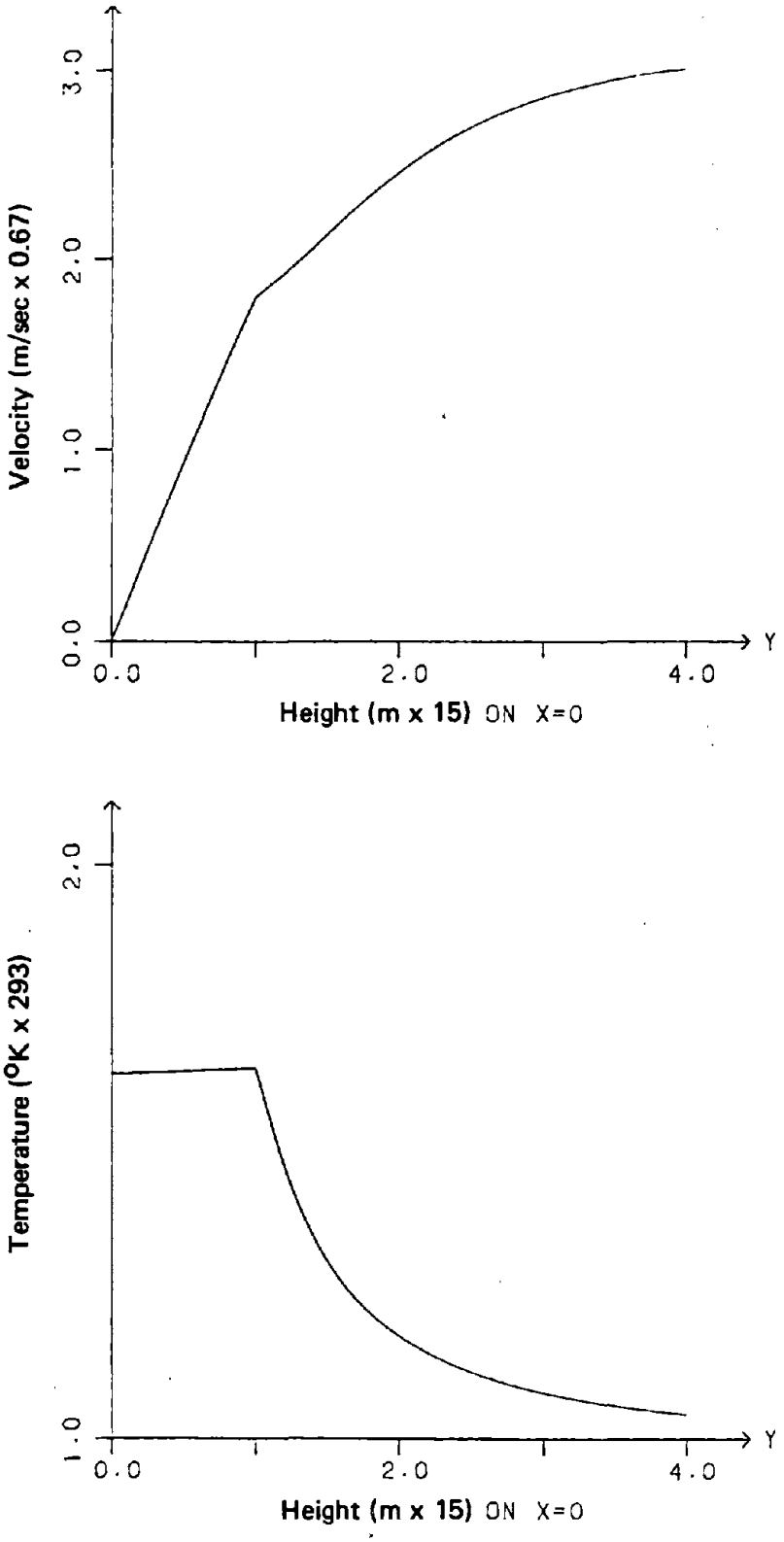


Fig. 46--Center-line velocities and temperatures predicted for typical multiple-fuel-bed Flambeau fire

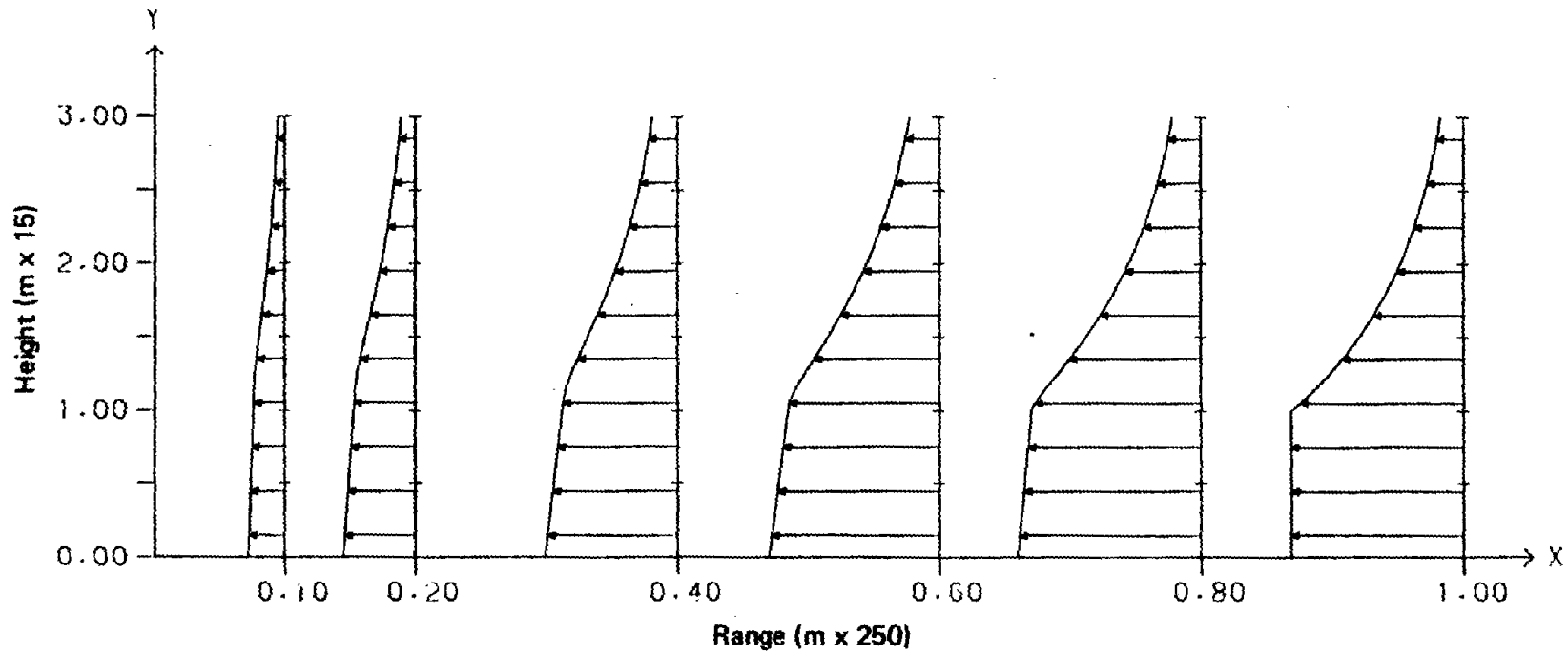


Fig. 47--Horizontal velocity profiles predicted for typical multiple-fuel-bed Flambeau fire

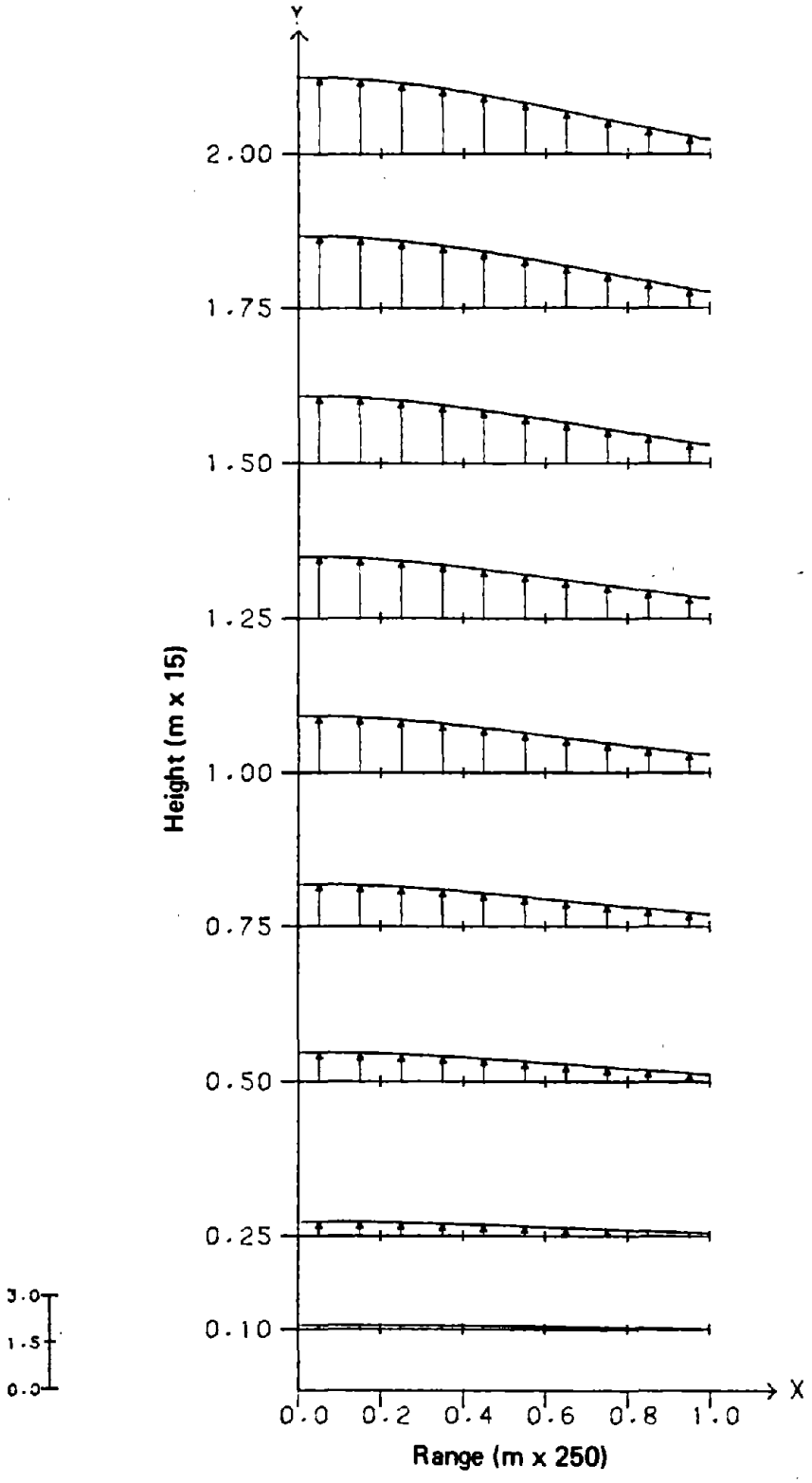


Fig. 48--Vertical velocity profiles predicted for typical multiple-fuel-bed Flambeau fire

IV. DISCUSSION

The use of a "component" model to describe large urban fires allows for detailed resolution of the dynamics of the combustion layer region without ignoring the influence of the rising plume or atmospheric recirculation. Ultimately, an even finer resolution of the combustion layer may be desired--for example, with a horizontal-length scale on the order of a city block, or even the width of a building. For such cases, the combustion layer model described here provides the ambient background against which the local flow (e.g., around buildings) and thermodynamic perturbations can be calculated. Our model thus represents a basic characterization of the environment in the fire zone.

The combustion layer model does have certain limitations. The combustive-heat release rate $Q \cdot q(x, y)$ must be prescribed independent of the dynamics; heat losses due to hot gas radiation are only qualitatively modeled, as if they were grey body or nearly "transparent" [Murgai, 1962]; and Reynolds stresses are only crudely approximated by diffusion terms with constant eddy diffusivities. In addition, as discussed in Sec. III, the proper velocity boundary condition along the outer edge of the fire and plume must still be determined.

More detailed modeling of the basic combustion processes, the resultant hot-gas radiation, and turbulence should, however, greatly reduce all limitations but the last. In general, those tasks are left for future investigations. Nevertheless, we note that Sec. III indicates the manner in which the combustion kinetics could approximately be coupled with the air flow. Our current simulation indicates that the sensitivity of the results to variations in the level of radiative cooling is relatively small and possibly negligible. Additionally, we anticipate that a more accurate description of the Reynolds stresses will show them to be much larger near the ground and near the symmetry axis than elsewhere, except possibly at the fire periphery.

More important, a refined combustion layer model can be used to answer major practical questions concerning the fire environment. One basic question concerns the dependence of the maximum fire-generated

winds on fire intensity [or, quantitatively, $\iint_{\mathcal{D}} Q \cdot q(x, y)\, dx\, dy$, with $\mathcal{D}$ denoting the fire zone]. Simple scaling provides a partial answer to this question: from Eq. (2.6), U is proportional to both Q and D. Such scaling provides order-of-magnitude estimates, but more detailed analysis is required for reliable quantitative predictions. For example, civil defense professionals should know whether the maximum wind speeds associated with the nominal scale in Eq. (2.7) of U ~ 150 mph are actually only 50 mph (U/3) or 450 mph (3U). A model like ours will provide such information, as well as predictions of corresponding temperature, concentrations of carbon monoxide and carbon dioxide, smoke maxima, and oxygen minima.

A second major practical question that can be resolved with model analysis concerns the horizontal extent of the induced fire winds and the corresponding range of wind damage. The solution entails the description of the fire-generated flow pattern near the bottom of the recirculation region (i.e., especially in subregions IA and IB of Fig. 3). One possibility is that the flow into the inlet region (IA) from regions III and IB is basically radial (i.e., sink-like), its magnitude falling off like $1/\hat{R}$, with

$$\hat{R} = \sqrt{\hat{x}^2 + y^2}\,, \qquad \hat{x} = \frac{x - 1}{\varepsilon^{\alpha}}\,. \tag{4.1}$$

In this case, the high fire winds would extend only slightly beyond the fire's periphery (i.e., only a horizontal distance on the order of $\varepsilon^{\alpha}D$). On the other hand, in the fully developed large-fire environment, the flow through regions IA and IB may well be nearly parallel. In that case, the fire winds would extend far past the fire periphery (and could be on the order of tens of kilometers for D ~ 10 km). The true situation can be determined only through further research. In fact, the answer to this question is probably coupled to the determination of the proper velocity boundary condition along the outer edge of the combustion layer--the nature of the flow in region IA (Fig. 3) being the key to both answers.

Even the current combustion layer model demonstrates one major physical observation: the width of the convection column can be comparable to that of the fire zone only if turbulence plays a dominant role in combustion layer dynamics. Otherwise, the column must be considerably thinner than the fire zone, with the air that passes through the fire converging toward the column center line and remaining near as it rises.

The two types of columns, which we refer to as thick and thin plumes, respectively, are illustrated in Fig. 49. The nature of the plume is a principal feature of any fire, and one that has been considered throughout the present investigation. Small free-burning fires exhibit a thin-plume structure in which the plume height is much greater than the cross-section dimension. Thin-plume fires have also been created in laboratory or table-top models with a closely spaced matrix of burners. In general, thin plumes can result from rotational or wind-shear effects, where centrifugal forces combined with a buoyant rise cause a fast-rising, rotating, thin column. Fuel storage fires for which the availability of the fuel for burning is dominated by the turbulent mixing of vapors with the external air often exhibit a thick plume.

All the Flambeau fires exhibited a thick plume, as did many of the large urban fires of World War II. For this reason, it is expected that the plumes over future urban fires will have horizontal dimensions comparable to the burning-fuel-bed dimension. That expectation is further supported by a more detailed analysis of the combustion in such fires. Blocks or houses will be burning furiously, with noncombustible streets or open areas in between. Such irregular local burning encourages a kind of localized turbulent action, which could then generate a thick-plume flow.

Additional numerical results not reported here confirm that the fire plume must be thin unless turbulence significantly affects the combustion layer dynamics. In those runs, the thickness of the plume tends toward zero as the diffusion coefficients are reduced, all other parameters (and boundary conditions) being held fixed.

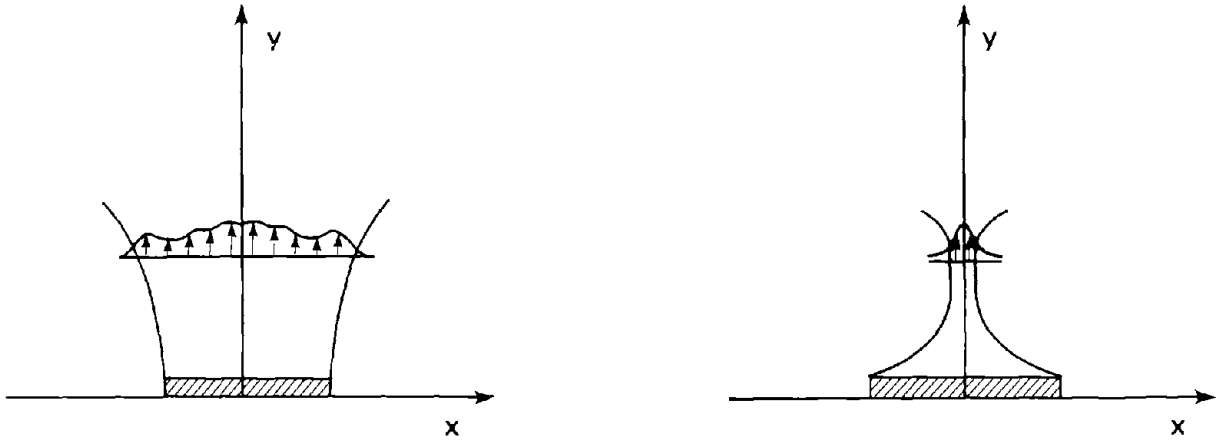


Fig. 49--Classification of possible convection plumes

Physically, this conclusion is explained as follows. In the combustion layer, the fire-generated buoyancy creates an updraft and a low pressure, so that air is sucked in from the far field. Those phenomena create large horizontal velocities, which must be decelerated if the flow is to be turned upward to form a thick plume. As demonstrated in the numerical runs, only in the region near the fire's center are the pressure gradients large enough to provide such a decelerating force. In fact, pressure gradients act over most of the combustion layer to accelerate the flow in toward the center. Other stresses--in particular, turbulent stresses--are thus required to provide deceleration and hence establish an upward thick-plume flow.

It is of major significance that our research provides the first description of the formation near the ground of a thick plume. A careful study of related work by other authors [Nielsen and Tao, 1965; Smith, Morton, and Leslie, 1975; Delage and Taylor, 1970 (for "heat island flows"--not fires)] shows that, in all cases, their results apply to thin plumes. Thus, though many more details of the gas dynamics of large burning regions can reasonably be incorporated into the calculational method, the current model reveals for the first time the nature of the thick plume and its relation to turbulent forces in and near the burning region.

In the modeling of both the combustion layer and the convection column flows, the effects of rotation (i.e., angular velocity) were for two reasons considered secondary and ignored. First, films of the multiple-fuel-bed Flambeau fires* showed no significant angular motion (swirl). Some slight rotation was observed in the initial build-up to a maximum heat release, and also in the final decay of the fires (in the form of small fire whirls), but neither transient situation pertains to the current modeling. Second, as shown in this report, a viable theory for a large area fire can be developed without including rotational phenomena.

Finally, research must be undertaken to determine the nature of the flow in the outflow region (IV) on top of the convection column

*From the U.S. Forest Service Laboratory, Riverside, California.

and the recirculation region. The modeling of that flow probably will involve additional meteorological physics (e.g., condensation, presence of inversion layers) not considered in the present analysis.

Large urban areas burning simultaneously are an expected result of nuclear warfare. The current modeling is intended to assist in understanding the nature of the velocity and temperature fields associated with such unprecedentedly large fires. If we succeed in modeling such fires, what have we learned? Of what value is the knowledge to civil defense? Effective efforts to limit damage under such extreme conditions must deal realistically with the actual environment that firefighting or rescue operations will face. Thus they would stand to profit from an improved understanding of the effects.

The possibility of additional damage due to large fire-associated or collateral winds can better be evaluated using the range of velocities predicted here. Concentrations of poisonous fumes, carbon monoxide, carbon dioxide, soot, and smoke can be calculated using the same mathematical model, and could be of great help in planning for adequate shelters and protection for key installations in an anticipated urban fire region. In short, the adequacy of any protective structures or actions against such large fires can better be evaluated using the calculations reported here.

Appendix A

SYMBOLS

MATHEMATICAL SYMBOLS

A, B = dimensionless constants

C_p = specific heat capacity at constant pressure

D = half-width of combustion zone

$\mathcal{D}$ = two-dimensional domain representing combustion zone

D_{1m}, D_{2m} = diffusion coefficients for species m, which represents oxygen, carbon monoxide, carbon dioxide, or smoke (particulates)

$\mathcal{E}_{11}, \mathcal{E}_{12}, \mathcal{E}_{21}, \mathcal{E}_{22}$ = effective kinematic viscosities (including turbulence effects)

$\mathfrak{F}$ = functional form for inlet velocity boundary condition

g = gravitational acceleration

$\hat{g}$ = dimensionless horizontal velocity inlet profile

H = scale height of atmosphere

$\mathcal{H}$ = functional form for inlet velocity boundary condition

k^* = reciprocal of grey-body radiation mean free path

k_1, k_2 = effective thermal conductivities (including turbulence effects)

K_1, K_2 = dimensionless heat diffusion coefficients

$\tilde{K}_1$ = rescaling of K_1

ℓ = mixing length

L = mean height of combustion zone

$M_{11}, M_{12}, M_{21}, M_{22}$ = dimensionless momentum diffusion coefficients

$\tilde{M}_{11}, \tilde{M}_{21}$ = rescalings of M_{11} and M_{21}

P = pressure

P_a = ground-level atmospheric pressure in far field

P_0 = leading-order pressure in $\varepsilon \to 0$ limit

P_i = correction pressure in $\varepsilon \to 0$ limit (integer i)

$\tilde{P}$ = sum of pressures through five orders in ε ($\varepsilon \to 0$ limit)

q = dimensionless volumetric heat-addition-rate distribution in combustion zone

Q = volume heat source scale

r = dimensionless radial coordinate

R = universal gas constant

$\hat{R}$ = dimensionless radial coordinate in the $(\hat{x}, y)$ plane

$\mathcal{R}_m$ = dimensionless volumetric depletion/addition rate distribution for species m, which represents oxygen, carbon monoxide, carbon dioxide, or smoke (particulates)

$\mathcal{S}_m$ = scales for volume depletion/addition during combustion of species m, which represents oxygen, carbon monoxide, carbon dioxide, or smoke (particulates)

T = temperature

T_a = ground-level atmospheric temperature in far field

T_0 = leading-order temperature in $\varepsilon \to 0$ limit

T_i = correction temperature in $\varepsilon \to 0$ limit (integer i)

T_∞, $\hat{T}_\infty$ = far-field ambient temperature in radiation law (dimensionless, dimensional)

$T_3^{(II)}$ = T_3 solution in region II

u = horizontal velocity

$\tilde{u}$ = rescaled horizontal velocity

u_0 = leading-order horizontal velocity in $\varepsilon \to 0$ limit

u_i = correction horizontal velocity in $\varepsilon \to 0$ limit (integer i)

$u_1^{(II)}$ = u_1 solution for region II

$u_{3/2}$ = correction horizontal velocity in $\varepsilon \to 0$ limit

$\hat{u}_{3/2}$ = relative horizontal velocity in side-mixing-layer region (IIA)

U_I = maximum inlet horizontal velocity in Flambeau fire of Fig. 4

U = horizontal velocity scale

v = vertical velocity

v_0 = leading-order vertical velocity in $\varepsilon \to 0$ limit

$v_0^{(II)}$ = v_0 solution in region II

v_i = correction vertical velocity in $\varepsilon \to 0$ limit (integer i)

v_∞ = vertical velocity versus x profile at top of combustion zone

$v_{1/2}$ = correction vertical velocity in $\varepsilon \to 0$ limit

x = horizontal position coordinate

$\tilde{x}$ = rescaled horizontal coordinate

$\hat{x}$ = rescaled horizontal coordinate

$\bar{x}$ = rescaled horizontal coordinate

y = vertical position coordinate

$\tilde{y}$ = rescaled vertical coordinate

y_∞ = maximum y value considered in numerical solution of leading-order combustion-layer problem

α = exponent such that the side-mixing-layer thickness is ε^α

$\hat{\alpha}$ = dimensionless parameter

β = dimensionless constant

$\hat{\beta}$, β_0 = dimensionless parameters

γ = specific heat ratio

Γ = dimensionless specific weight

Γ_∞ = dimensionless far-field ambient specific weight

δ_1, δ_2 = dimensionless constants

ε = combustion-zone aspect ratio

η = horizontal coordinate of the side of the convection column (as a function of height)

ρ = density

ρ_a = ground-level atmospheric density in far field

ρ_0 = leading-order density in $\varepsilon \to 0$ limit

ρ_i = correction density in $\varepsilon \to 0$ limit (integer i)

ρ_∞ = total far-field ambient density through five orders in ε ($\varepsilon \to 0$ limit)

σ = Stefan's constant

$\hat{\sigma}$ = dimensionless constant

ϕ_m = mass fraction of species m in air, where m represents oxygen, carbon monoxide, carbon dioxide, or smoke (particulates)

$\hat{\phi}$ = derivative with respect to y of the dimensionless inlet vertical velocity

ω = relaxation factor in overall FIRE/FIRE2 code iteration

SYMBOLS* USED IN DOCUMENTATION OF THE COMBUSTION LAYER ALGORITHM

$\hat{A}$ = product of A and B

$\vec{F}$, $\vec{G}$, $\vec{H}$ = vectors of functions that are to be driven to zero (by the proper choice of $\vec{x}$) in Newton iterations, and that represent discretized model equations on individual lines of constant y

*Mathematical variables only--not FORTRAN labels.

G_i, H_i = i*th* components of $\vec{G}$ and $\vec{H}$

i = horizontal position index ($1 \leq i \leq M + 1$)

j = vertical position index ($1 \leq j \leq N + 1$)

$J = \frac{\partial \vec{F}}{\partial \vec{x}}$ Jacobian matrix

M, N = number of finite-difference cells in the horizontal and vertical directions

$\bar{M}$ = $(M + 1)$ for condition in Eq. (C.3), and M for Eq. (C.4)

$\hat{P}_1$ = rescaling of P_1

$P_{i,j} = \hat{P}_1(x_i, y_j)$

R_i = sum of all terms in G_i for any line $y = y_j$, j fixed, that depend only on $y = y_{j-1}$ data

$\vec{T}$ = vector of discrete T_0 values for a line of constant y

$T_{i,j} = T_0(x_i, y_j)$

$\vec{u}$ = vector of discrete u_0 values for a line of constant y

$u_{i,j} = u_0(x_i, y_j)$

$v_{i,j} = v_0(x_i, y_j)$

$w_{i,j}$ = generic finite-difference variable (for all i, j)

$\vec{x}$ = vector of discrete u_0 and T_0 values for a line of constant y

$\tilde{x}_i$ = i*th* component of $\vec{x}$

$x_i = (i - 1)(\Delta x)$

$y_j = (j - 1)(\Delta y)$

$\alpha_{i,j}$ = nonzero element in the $\frac{\partial \vec{G}}{\partial \vec{u}}$ Jacobian ($0 \leq i \leq 2$, $1 \leq j \leq \bar{M} - 1$)

$\beta_{i,j}$ = nonzero element in the $\frac{\partial \vec{G}}{\partial \vec{T}}$ Jacobian ($0 \leq i \leq 2$, $1 \leq j \leq M - 1$)

$\gamma_{i,j}$ = nonzero element in the $\frac{\partial \vec{H}}{\partial \vec{u}}$ Jacobian ($0 \leq i \leq 2$, $1 \leq j \leq \bar{M} - 1$)

$\delta_{i,j}$ = nonzero element in the $\frac{\partial \vec{H}}{\partial T}$ Jacobian ($0 \leq i \leq 2$, $1 \leq j \leq M - 1$)

$\vec{\delta}$ = vector difference between successive values of $\vec{x}$ in Newton iterations

Δx, Δy = horizontal and vertical mesh widths

II_i = sum of all terms in H_i for any line $y = y_j$, j fixed, that depend only on $y = y_{j-1}$ data

Appendix B

ANALYTIC DEVELOPMENT FOR CONVECTION COLUMN MODEL

The order estimates in Eq. (2.18) result directly from using the conventional mixing-length approximation

$$\frac{\mathcal{E}_{11}}{\rho_a} , \frac{\mathcal{E}_{12}}{\rho_a} , \frac{K_1}{\rho_a} \sim \ell^2 \left(\frac{\partial v}{\partial x}\right) , \tag{B.1}$$

where ℓ is the mixing length. Subject to the parameter definitions in Eq. (2.4) and the scalings in Eq. (2.2), then (for j = 1, 2) in nondimensional variables

$$\frac{M_{ij}}{\epsilon} , \frac{K_1}{\epsilon} \sim \left(\frac{1}{U\epsilon D}\right)\left[\ell^2 \left(\frac{\epsilon U}{D}\right)\left(\frac{\partial v}{\partial x}\right)\right] \sim \left(\frac{\ell}{D}\right)^2 . \tag{B.2}$$

Recalling that $D \sim 10^4$ m and assuming a mixing length $\ell \lesssim 10^3$ m, we obtain the first part of Eq. (2.18). The second part then follows from the fact that the horizontal turbulence effects must dominate for the basically upward column flow.

The leading-order equations in Eq. (2.28) for the side mixing layer are derived as follows. Subject to Eq. (2.18) and the rescaling in Eq. (2.27), Eq. (2.17) becomes

$$\left(\frac{1}{\sqrt{\epsilon}}\right)\left[\frac{\partial}{\partial \tilde{x}} (\rho \tilde{u}) - \left(\frac{d\eta}{d\tilde{y}}\right)\frac{\partial}{\partial \tilde{x}} (\rho v)\right] + \frac{\partial}{\partial \tilde{y}} (\rho v) = 0 ;$$

$$\rho\left\{\left(\frac{1}{\sqrt{\epsilon}}\right)\left[\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} - \left(\frac{d\eta}{d\tilde{y}}\right) v \frac{\partial u}{\partial \tilde{x}}\right] + v\frac{\partial \tilde{u}}{\partial \tilde{y}}\right\} = -\frac{B}{\epsilon^{7/2}}\frac{\partial P}{\partial \tilde{x}} + \left(\frac{M_{11}}{\epsilon^2}\right)\left[\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + o(1)\right] ;$$

$$\rho\left\{\left(\frac{1}{\sqrt{\epsilon}}\right)\left[\tilde{u}\frac{\partial v}{\partial \tilde{x}} - \left(\frac{d\eta}{d\tilde{y}}\right) v \frac{\partial v}{\partial \tilde{x}}\right] + v\frac{\partial v}{\partial \tilde{y}}\right\} = -\frac{B}{\epsilon^3}\left(\frac{\partial P}{\partial \tilde{y}} + A\rho\right) + \left(\frac{M_{21}}{\epsilon^2}\right)\left[\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + o(1)\right] ;$$

$$\rho\left\{\left(\frac{1}{\sqrt{\epsilon}}\right)\left[\tilde{u}\,\frac{\partial T}{\partial \tilde{x}} - \left(\frac{d\eta}{d\tilde{y}}\right) v\,\frac{\partial T}{\partial \tilde{x}}\right] + v\,\frac{\partial T}{\partial \tilde{y}}\right\} = \left(\frac{\gamma - 1}{\gamma}\right)\left\{\left(\frac{1}{\sqrt{\epsilon}}\right)\left[\tilde{u}\,\frac{\partial P}{\partial \tilde{x}} - \left(\frac{d\eta}{d\tilde{y}}\right) v\,\frac{\partial P}{\partial \tilde{x}}\right] + v\,\frac{\partial P}{\partial \tilde{y}}\right\} + \left(\frac{K_1}{\epsilon^2}\right)\left[\frac{\partial^2 T}{\partial \tilde{x}^2} + o(1)\right] ;$$

$$P = \rho T \ . \tag{B.3}$$

Substituting the expansions for $\tilde{u}$ and v in Eq. (2.29) and those for P, ρ, and T in Eq. (2.10) into Eq. (B.3), it is found that ρ_0, T_0, P_0, ρ_1, T_1, P_1, ρ_2, T_2, and P_2 are all functions of $\tilde{y}$ alone, and hence (from the continuity equation)

$$\rho_0(\tilde{y})\,\frac{\partial}{\partial \tilde{x}}\left[u_1 - \left(\frac{d\eta}{d\tilde{y}}\right) v_0\right] = 0 \ . \tag{B.4}$$

The quantity $[u_1 - (\frac{d\eta}{d\tilde{y}})\, v_0]$ is thus also a function of $\tilde{y}$ alone and, in particular, must (for a matching of region II and IIA solutions) be

$$\lim_{x \to \eta(\tilde{y})} \left[u_1 - \left(\frac{d\eta}{d\tilde{y}}\right) v_0\right]_{\text{region II}} \ .$$

But this limit must actually be zero for all $\tilde{y}$, since the $x = \eta(\tilde{y})$ streamline is defined by

$$\frac{d\eta}{d\tilde{y}} = \lim_{x \to \eta(\tilde{y})} \left[\left(\frac{u_1}{v_0}\right)\right]_{\text{region II}} \ .$$

Thus

$$u_1 = \left(\frac{d\eta}{d\tilde{y}}\right) v_0 \ , \tag{B.5}$$

and the other leading-order equations in region II become [using Eqs. (2.10) and (2.29) in Eq. (B.3)]

$$\frac{\partial P_3}{\partial \tilde{x}} = 0 \ ;$$

$$\rho_0 \left\{ \left[u_{3/2} - \left(\frac{d\eta}{d\tilde{y}} \right) v_{1/2} \right] \frac{\partial v_0}{\partial \tilde{x}} + v_0 \frac{\partial v_0}{\partial \tilde{y}} \right\} = - B \left(\frac{\partial P_3}{\partial \tilde{y}} + A\rho \right) + \tilde{M}_{21} \frac{\partial^2 v_0}{\partial \tilde{x}^2} \ ;$$

$$\rho_0 \left\{ \left[u_{3/2} - \left(\frac{d\eta}{d\tilde{y}} \right) v_{1/2} \right] \frac{\partial T_3}{\partial \tilde{x}} + v_0 \frac{\partial T_3}{\partial \tilde{y}} \right\} = \left(\frac{\gamma - 1}{\gamma} \right) \left\{ \left[u_{3/2} - \left(\frac{d\eta}{d\tilde{y}} \right) v_{1/2} \right] \frac{\partial P_3}{\partial \tilde{x}} + v_0 \frac{\partial P_3}{\partial \tilde{y}} \right\}$$

$$+ \tilde{K}_1 \frac{\partial^2 T_3}{\partial \tilde{x}^2} - v_0 \left(\rho_1 \frac{dT_2}{d\tilde{y}} + \rho_2 \frac{dT_1}{d\tilde{y}} + \rho_3 \frac{dT_0}{d\tilde{y}} \right) \ ;$$

$$P_3 = \rho_0 T_3 + \rho_1 T_2 + \rho_2 T_1 + \rho_3 T_0 \ , \qquad \text{(B.6)}$$

with $\tilde{M}_{21}$ and $\tilde{K}_1$ as defined in Eq. (2.31). Finally, the first correction to the leading-order continuity equation [Eq. (B.4)] is then

$$\frac{\partial}{\partial \tilde{x}} \left\{ \rho_0 \left[u_{3/2} - \left(\frac{d\eta}{d\tilde{y}} \right) v_{1/2} \right] \right\} + \frac{\partial}{\partial \tilde{y}} (\rho_0 v_0) = 0 \ . \qquad \text{(B.7)}$$

Equations (B.4) and (B.7) and the final three equations in Eq. (B.6) [with $(\partial P_3/\partial \tilde{x}) = 0$] are equivalent to Eq. (2.28) once the definition of Eq. (2.30) is made.

Appendix C

ALGORITHM DOCUMENTATION

The construction of the solution algorithm for the combustion-layer boundary-value problem depends on the form of the inlet boundary condition [cf. Eqs. (3.5) and (3.6)]. In the general case [Eq. (3.5)], an additional difference equation is required on each line of constant y, necessitating an expanded variable matrix. For convenience, separate codes were written to account for the slight differences in logic. Both algorithms provide numerical solutions to

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 \ ;$$

$$\rho_0 \left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) = - \frac{\partial \hat{P}_1}{\partial x} + M_{11} \frac{\partial^2 u_0}{\partial x^2} \ ;$$

$$\frac{\partial \hat{P}_1}{\partial y} + \hat{A}\rho_0 = 0 \ ;$$

$$\rho_0 \left(u_0 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_0}{\partial y} \right) = q(x, y) - \sigma(T_0^4 - 1) + K_1 \frac{\partial^2 T_0}{\partial x^2} \ ;$$

$$\rho_0 T_0 = 1 \ , \qquad \text{(C.1)}$$

subject to

$$\begin{aligned} &\text{a.} \quad \text{along } y = 0 \ , && v_0 = 0 \ , \\ &\text{b.} \quad \text{along } x = 0 \ , && u_0 = \frac{\partial T_0}{\partial x} = 0 \ , \\ &\text{c.} \quad \text{as } y \to \infty \ , && \frac{\partial \hat{P}_1}{\partial x} \to 0 \ , \\ &\text{d.} \quad \text{along } x = 1 \ , && T_0 = 1 \ , \end{aligned} \qquad \text{(C.2)}$$

with different versions for

$$\text{i. along } x = 1 , \qquad \frac{\partial}{\partial x}(\rho_0 u_0) = \hat{\phi}(y) + \beta_0 u , \tag{C.3}$$

$$\text{ii. along } x = 1 , \qquad u_0 = \hat{g}(y) . \tag{C.4}$$

For convenience, $P_1(x, y)$ and A have been rescaled as

$$\hat{P}_1 = BP_1 ; \qquad \hat{A} = BA . \tag{C.5}$$

As shown in the flowchart in Fig. 6 (p. 35), the codes use an iterative "shooting" method [Keller, 1968] to solve the overall boundary value problem, each "shoot" employing a nonlinear Crank-Nicolson finite-difference scheme.

INPUTS AND OUTPUTS

The following is an ordered list of the (FORTRAN) inputs for the code corresponding to Eq. (C.3):

a. System parameters:

A	$\hat{A}$ [in Eq. (C.5)]
SIGMA	σ
XNUIN	M_{11}
XKIN	K_1

b. Mesh descriptors:

YMAX	y_∞
M	Number of finite difference cells in horizontal direction.*
N	Number of finite difference cells in vertical direction.*

c. Iteration parameters:

ETLINE	Maximum absolute error allowed in solving discretized (Crank-Nicolson) equations on any line of constant y.
ETOLP	Maximum absolute error allowed in $\partial\hat{P}_1/\partial x$ on $y = y_\infty$.

*Mesh widths and heights are individual constants.

MAXITL — Maximum number of (Newton) iterations allowed in solving discretized equations on any line of constant y.

MAXITP — Maximum number of (complete shoot) iterations allowed in solving the complete boundary value problem.

d. Boundary condition parameters:

ALPHA/BETA — $\hat{\alpha}$ and $\hat{\beta}$, parameters that define $\hat{\phi}(y)$ as

$$\hat{\phi}(y) = -\hat{\beta}\ \mathrm{sech}^2\left(\frac{\hat{\beta}}{\hat{\alpha}}\right) y \qquad \text{(C.6)}$$

for the case $\beta_0 = 0$ and $v_0(1, y)$ as specified in Eq. (3.4) [cf. Eq. (3.6)].

BLOSS — β_0

IGUESS — Parameter that controls the specification of the (first shoot, y = 0) initial guesses for IGUESS = 1, 2, or 3:

1. Discretized versions of $\hat{P}_1(x, 0)$, $T_0(x, 0)$, and $u_0(x, 0)$ must also be supplied as inputs.
2. A discretized version of $\hat{P}_1(x, 0)$ must also be supplied, but "standard" specifications of $T_0(x, 0)$ and $u_0(x, 0)$ are automatically made. ("Standard" specifications may of course be modified by reprogramming.)
3. "Standard" initial guesses are made for all variables.

Additionally, the functional form of q(x, y) must be prescribed. Currently, this is given by

$$q(x, y) = \left\{ \begin{array}{l} 1 \text{ for } 0 \le x \le 1\ , \quad 0 \le y \le 1 \\ \\ 0 \text{ otherwise} \end{array} \right\} . \qquad \text{(C.7)}$$

Changes require reprogramming (in subroutine BCFUNC). Finally, as an added option, the dissipation coefficients M_{11} and K_1 may also be made to depend on x and y via reprogramming (also in BCFUNC). Currently, those coefficients are simply taken as constants.

A sample input data stream is

A = 0.16 , SIGMA = 0.023 , XNUIN = XKIN = 0.5 ;

YMAX = 6 , M = 40 , N = 400 ;

ETLINE = 0.001 , ETOLP = 0.005 , MAXITL = MAXITP = 20 ;

ALPHA = 1.0 , BETA = 1.5 , BLOSS = 0.0 , IGUESS = 2 ;

with $\hat{\phi}(y)$ as in Eq. (C.6). The corresponding output is listed in Table C.1. With IGUESS = 2, $\hat{P}_1(x, 0)$, data similar to those in the J = 1 (y = 0.0) printout were used to generate the output. The other, "standard" initial function guesses were

$$T_0(x, 0) = 1.76 - 0.6x, \; u_0(x, 0) = 1.9 \tanh (1.6x) .$$

Additionally, all output data are automatically dumped onto a disk file from which are made the vector, contour, and profile plots displayed in Sec. III.

Inputs and corresponding outputs for both codes are almost identical. The only change is that the functional form of u(1, y) must be prescribed (in subroutine BCFUNC) when Eq. (C.4) is used, whereas ALPHA, BETA, and BLOSS are used to describe $\frac{\partial}{\partial x} (\rho_0 u_0)$ when Eq. (C.3) is used.

PROGRAM FLOW

A macro flowchart for both codes is given in Fig. 6 (p. 35). A flowchart for the main program is presented in Fig. C.1. Inputs/outputs and the overall shooting iteration are controlled by the main program. In subroutine BCFUNC, functional forms that may be varied [e.g., q(x, y)] are specified; and in subroutine SPRINT, final output data are printed. In subroutine MSWEEP, most of the computations--those providing a (one shoot) finite difference solution to Eq. (3.1) for given data along y = 0, x = 0, and x = 1 (only)--are carried out. The flow of that subroutine is shown in Figs. C.3 and C.4 below (pp. 124-125) and details of the underlying numerical analysis presented in the accompanying text. Complete FORTRAN listings of all subroutines and main programs are presented at the end of this appendix.

Table C.1

SAMPLE FIRE CODE OUTPUT

FIRE: SOLUTION OF COMBUSTION LAYER BOUNDARY VALUE PROBLEM

SYSTEM PARAMETERS: A = 0.160 SIGMA = 0.023 NU = 0.500 K = 0.500

MESH: M = 40 DX = 0.025 N = 400 DY = 0.025

ITERATION PARAMETERS: ETLINE = 0.001000 MAXITL = 20 ETOLP = 0.005000 MAXITP = 20

INLET PARAMETERS: ALPHA = 1.000 BETA = 1.500 BLOSS = 0.000

FOR J = 1 (Y = 0.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.000	1.519	0.658	-0.155	0.000	0.000	0.000	0.900
2	0.025	-0.083	0.000	1.519	0.658	-0.155	0.004	0.185	-0.899	0.900
3	0.050	-0.165	0.000	1.518	0.659	-0.155	0.015	0.374	-0.894	0.901
4	0.075	-0.248	0.000	1.516	0.660	-0.155	0.024	0.558	-0.883	0.901
5	0.100	-0.329	0.000	1.513	0.661	-0.154	0.033	0.737	-0.869	0.903
6	0.125	-0.410	0.000	1.508	0.663	-0.153	0.043	0.912	-0.851	0.904
7	0.150	-0.489	0.000	1.503	0.665	-0.152	0.054	1.079	-0.826	0.906
8	0.175	-0.567	0.000	1.496	0.668	-0.150	0.062	1.233	-0.802	0.908
9	0.200	-0.643	0.000	1.489	0.672	-0.149	0.071	1.378	-0.771	0.910
10	0.225	-0.718	0.000	1.480	0.676	-0.147	0.081	1.515	-0.739	0.913
11	0.250	-0.791	0.000	1.471	0.680	-0.145	0.089	1.635	-0.703	0.915
12	0.275	-0.862	0.000	1.461	0.685	-0.142	0.097	1.743	-0.666	0.918
13	0.300	-0.931	0.000	1.449	0.690	-0.140	0.107	1.841	-0.625	0.921
14	0.325	-0.997	0.000	1.437	0.696	-0.137	0.117	1.925	-0.584	0.925
15	0.350	-1.061	0.000	1.425	0.702	-0.134	0.124	1.991	-0.542	0.928
16	0.375	-1.122	0.000	1.412	0.708	-0.131	0.131	2.045	-0.497	0.932
17	0.400	-1.181	0.000	1.398	0.716	-0.127	0.138	2.087	-0.455	0.935
18	0.425	-1.238	0.000	1.383	0.723	-0.124	0.147	2.114	-0.410	0.939
19	0.450	-1.291	0.000	1.368	0.731	-0.120	0.154	2.129	-0.366	0.942
20	0.475	-1.342	0.000	1.353	0.739	-0.116	0.160	2.131	-0.323	0.946
21	0.500	-1.391	0.000	1.337	0.748	-0.112	0.169	2.126	-0.282	0.950
22	0.525	-1.436	0.000	1.320	0.757	-0.108	0.175	2.103	-0.240	0.953
23	0.550	-1.479	0.000	1.304	0.767	-0.103	0.182	2.077	-0.201	0.957
24	0.575	-1.520	0.000	1.287	0.777	-0.099	0.188	2.038	-0.162	0.960
25	0.600	-1.558	0.000	1.270	0.787	-0.094	0.194	1.991	-0.127	0.963
26	0.625	-1.593	0.000	1.253	0.798	-0.089	0.201	1.940	-0.091	0.966
27	0.650	-1.626	0.000	1.236	0.809	-0.084	0.204	1.879	-0.059	0.969
28	0.675	-1.657	0.000	1.218	0.821	-0.079	0.211	1.814	-0.029	0.972
29	0.700	-1.685	0.000	1.201	0.833	-0.073	0.218	1.749	-0.001	0.975
30	0.725	-1.711	0.000	1.184	0.845	-0.068	0.220	1.675	0.026	0.978
31	0.750	-1.735	0.000	1.166	0.857	-0.062	0.225	1.598	0.050	0.980
32	0.775	-1.757	0.000	1.149	0.870	-0.057	0.231	1.525	0.070	0.983
33	0.800	-1.778	0.000	1.132	0.883	-0.051	0.238	1.449	0.092	0.985
34	0.825	-1.796	0.000	1.115	0.897	-0.045	0.240	1.368	0.109	0.987
35	0.850	-1.813	0.000	1.098	0.911	-0.039	0.244	1.292	0.124	0.990
36	0.875	-1.828	0.000	1.081	0.925	-0.033	0.252	1.217	0.138	0.992
37	0.900	-1.841	0.000	1.064	0.939	-0.026	0.256	1.143	0.149	0.993
38	0.925	-1.853	0.000	1.048	0.954	-0.020	0.258	1.066	0.160	0.995
39	0.950	-1.864	0.000	1.032	0.969	-0.013	0.261	0.994	0.167	0.997
40	0.975	-1.874	0.000	1.016	0.984	-0.007	0.267	0.927	0.175	0.999
41	1.000	-1.882	0.000	1.000	1.000	0.000	0.270	0.927	0.175	1.000

Table C.1--Continued

FOR J = 2 (Y = 0.025):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	0.083	1.520	0.658	-0.158	0.000	0.000	0.000	0.900
2	0.025	-0.082	0.083	1.520	0.658	-0.158	0.004	0.183	-0.896	0.900
3	0.050	-0.165	0.082	1.519	0.659	-0.158	0.015	0.371	-0.890	0.901
4	0.075	-0.246	0.082	1.516	0.660	-0.157	0.024	0.553	-0.880	0.901
5	0.100	-0.327	0.082	1.513	0.661	-0.157	0.032	0.731	-0.867	0.903
6	0.125	-0.407	0.081	1.509	0.663	-0.156	0.043	0.903	-0.846	0.904
7	0.150	-0.486	0.080	1.503	0.665	-0.155	0.054	1.069	-0.828	0.906
8	0.175	-0.564	0.080	1.497	0.668	-0.153	0.062	1.222	-0.798	0.908
9	0.200	-0.640	0.079	1.489	0.672	-0.151	0.070	1.366	-0.770	0.910
10	0.225	-0.714	0.078	1.481	0.675	-0.150	0.080	1.501	-0.738	0.913
11	0.250	-0.787	0.077	1.471	0.680	-0.147	0.088	1.621	-0.703	0.915
12	0.275	-0.857	0.076	1.461	0.685	-0.145	0.096	1.729	-0.664	0.918
13	0.300	-0.926	0.075	1.450	0.690	-0.143	0.106	1.825	-0.627	0.921
14	0.325	-0.992	0.073	1.438	0.695	-0.140	0.116	1.908	-0.584	0.925
15	0.350	-1.055	0.072	1.425	0.702	-0.137	0.123	1.978	-0.542	0.928
16	0.375	-1.116	0.071	1.412	0.708	-0.134	0.130	2.029	-0.500	0.932
17	0.400	-1.175	0.069	1.398	0.715	-0.130	0.137	2.071	-0.455	0.935
18	0.425	-1.231	0.068	1.384	0.723	-0.127	0.146	2.100	-0.413	0.939
19	0.450	-1.284	0.067	1.369	0.731	-0.123	0.152	2.116	-0.368	0.942
20	0.475	-1.335	0.065	1.353	0.739	-0.119	0.159	2.119	-0.327	0.946
21	0.500	-1.383	0.064	1.337	0.748	-0.115	0.167	2.110	-0.284	0.950
22	0.525	-1.429	0.062	1.321	0.757	-0.111	0.173	2.094	-0.243	0.953
23	0.550	-1.472	0.061	1.305	0.767	-0.106	0.181	2.065	-0.203	0.956
24	0.575	-1.512	0.059	1.288	0.777	-0.102	0.187	2.028	-0.166	0.960
25	0.600	-1.550	0.057	1.271	0.787	-0.097	0.192	1.983	-0.128	0.963
26	0.625	-1.585	0.056	1.254	0.798	-0.092	0.199	1.931	-0.095	0.966
27	0.650	-1.618	0.054	1.236	0.809	-0.087	0.202	1.870	-0.063	0.969
28	0.675	-1.648	0.053	1.219	0.820	-0.082	0.209	1.810	-0.031	0.972
29	0.700	-1.677	0.052	1.202	0.832	-0.077	0.216	1.743	-0.003	0.975
30	0.725	-1.703	0.050	1.184	0.844	-0.071	0.218	1.668	0.021	0.978
31	0.750	-1.727	0.049	1.167	0.857	-0.066	0.223	1.597	0.047	0.980
32	0.775	-1.749	0.047	1.150	0.870	-0.060	0.229	1.520	0.068	0.983
33	0.800	-1.769	0.046	1.132	0.883	-0.054	0.235	1.447	0.088	0.985
34	0.825	-1.787	0.045	1.115	0.897	-0.048	0.237	1.366	0.105	0.987
35	0.850	-1.804	0.044	1.098	0.911	-0.042	0.241	1.289	0.122	0.990
36	0.875	-1.819	0.042	1.081	0.925	-0.036	0.249	1.218	0.134	0.992
37	0.900	-1.832	0.041	1.065	0.939	-0.030	0.253	1.141	0.148	0.993
38	0.925	-1.844	0.040	1.048	0.954	-0.024	0.255	1.067	0.156	0.995
39	0.950	-1.855	0.039	1.032	0.969	-0.017	0.259	0.996	0.166	0.997
40	0.975	-1.864	0.038	1.016	0.984	-0.011	0.264	0.926	0.173	0.999
41	1.000	-1.873	0.037	1.000	1.000	-0.004	0.267	0.926	0.173	1.000

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Table C.1--Continued

FOR J = 401 (Y = 10.000):

I	X	U	V	T	RHO	P	PX	NU*UXX	K*TXX	Q(T)
1	0.000	0.000	3.525	1.004	0.996	-1.600	0.000	0.000	0.000	0.000
2	0.025	0.000	3.521	1.004	0.996	-1.600	0.000	0.000	-0.006	0.000
3	0.050	0.000	3.511	1.004	0.996	-1.600	0.001	0.002	-0.005	0.000
4	0.075	-0.001	3.495	1.004	0.996	-1.600	0.000	0.001	-0.006	0.000
5	0.100	-0.001	3.473	1.004	0.996	-1.600	-0.001	0.001	-0.006	0.000
6	0.125	-0.001	3.446	1.004	0.996	-1.600	0.000	0.002	-0.005	0.000
7	0.150	-0.001	3.413	1.004	0.996	-1.600	0.002	0.004	-0.005	0.000
8	0.175	-0.001	3.375	1.004	0.996	-1.600	0.001	0.003	-0.006	0.000
9	0.200	-0.001	3.331	1.004	0.996	-1.600	0.000	0.003	-0.005	0.000
10	0.225	-0.002	3.283	1.004	0.996	-1.600	0.001	0.004	-0.005	0.000
11	0.250	-0.002	3.230	1.004	0.996	-1.600	0.000	0.004	-0.005	0.000
12	0.275	-0.002	3.174	1.004	0.996	-1.600	-0.001	0.003	-0.005	0.000
13	0.300	-0.002	3.113	1.003	0.997	-1.600	0.001	0.005	-0.005	0.000
14	0.325	-0.002	3.047	1.003	0.997	-1.600	0.002	0.006	-0.004	0.000
15	0.350	-0.002	2.978	1.003	0.997	-1.600	0.001	0.005	-0.005	0.000
16	0.375	-0.002	2.906	1.003	0.997	-1.600	0.000	0.005	-0.005	0.000
17	0.400	-0.003	2.832	1.003	0.997	-1.600	0.000	0.004	-0.003	0.000
18	0.425	-0.003	2.755	1.003	0.997	-1.600	0.001	0.006	-0.003	0.000
19	0.450	-0.003	2.676	1.003	0.997	-1.600	0.000	0.005	-0.004	0.000
20	0.475	-0.003	2.595	1.003	0.997	-1.600	0.000	0.004	-0.002	0.000
21	0.500	-0.003	2.512	1.003	0.997	-1.600	0.001	0.006	-0.004	0.000
22	0.525	-0.003	2.428	1.003	0.997	-1.600	0.000	0.005	-0.002	0.000
23	0.550	-0.003	2.343	1.002	0.998	-1.600	0.001	0.006	-0.002	0.000
24	0.575	-0.003	2.258	1.002	0.998	-1.600	0.001	0.005	-0.002	0.000
25	0.600	-0.003	2.172	1.002	0.998	-1.600	0.000	0.005	-0.002	0.000
26	0.625	-0.003	2.086	1.002	0.998	-1.600	0.002	0.006	-0.002	0.000
27	0.650	-0.003	2.000	1.002	0.998	-1.600	-0.001	0.003	-0.002	0.000
28	0.675	-0.003	1.916	1.002	0.998	-1.600	0.000	0.005	0.000	0.000
29	0.700	-0.004	1.831	1.002	0.998	-1.600	0.002	0.006	-0.002	0.000
30	0.725	-0.004	1.748	1.002	0.998	-1.600	-0.001	0.003	-0.002	0.000
31	0.750	-0.004	1.667	1.001	0.999	-1.600	-0.001	0.003	0.001	0.000
32	0.775	-0.004	1.587	1.001	0.999	-1.600	0.000	0.004	-0.001	0.000
33	0.800	-0.004	1.508	1.001	0.999	-1.600	0.001	0.005	-0.002	0.000
34	0.825	-0.004	1.431	1.001	0.999	-1.600	-0.001	0.002	0.001	0.000
35	0.850	-0.004	1.356	1.001	0.999	-1.600	-0.002	0.002	-0.003	0.000
36	0.875	-0.004	1.283	1.001	0.999	-1.600	0.002	0.005	0.002	0.000
37	0.900	-0.004	1.211	1.001	0.999	-1.600	0.001	0.004	-0.002	0.000
38	0.925	-0.004	1.142	1.000	1.000	-1.600	-0.002	0.001	0.002	0.000
39	0.950	-0.004	1.076	1.000	1.000	-1.600	-0.002	0.000	-0.004	0.000
40	0.975	-0.004	1.013	1.000	1.000	-1.600	-0.001	0.001	0.004	0.000
41	1.000	-0.004	0.961	1.000	1.000	-1.600	0.001	0.001	0.004	0.000

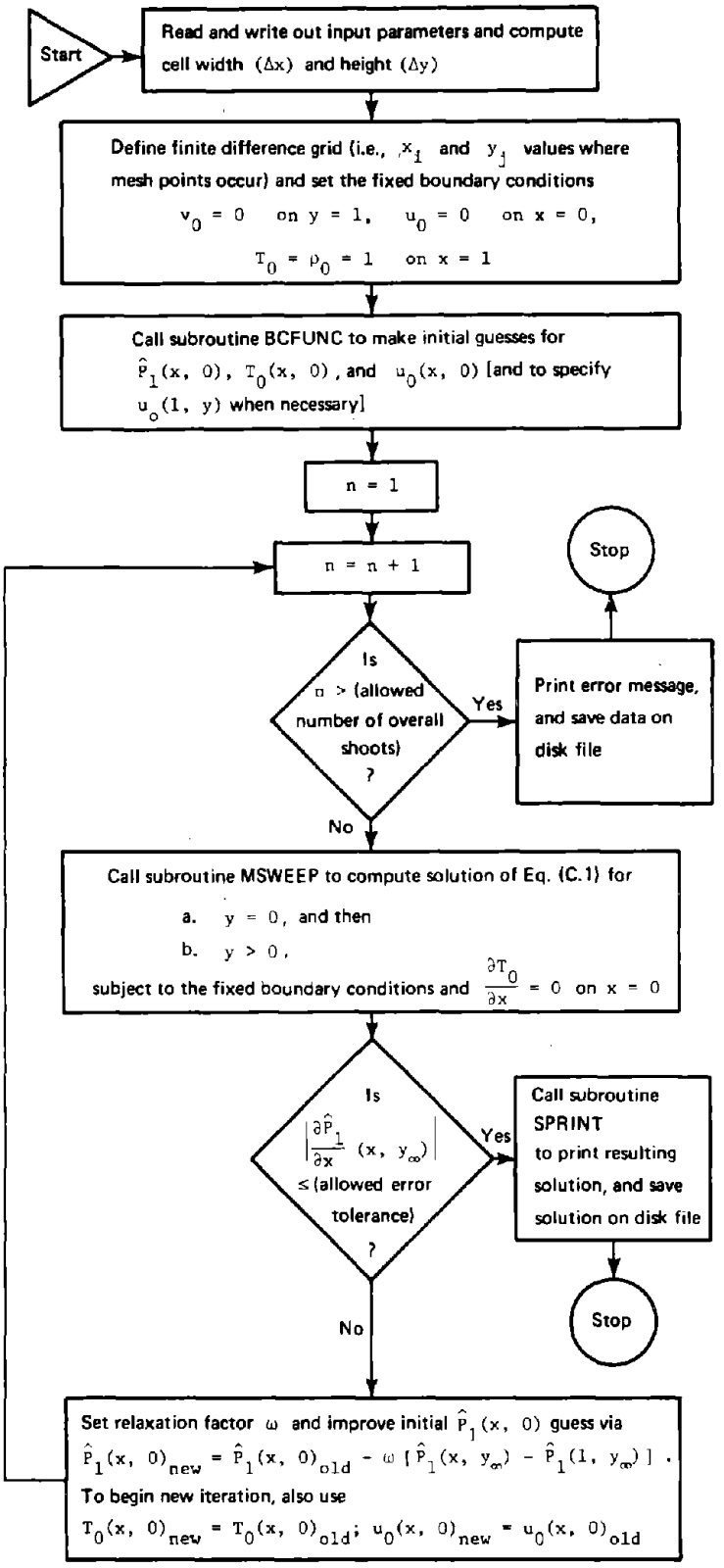

Fig. C.1--Flowchart of main program

FINITE DIFFERENCE ANALYSIS AND SUBROUTINE MSWEEP

The finite difference scheme underlying both codes is based on the rectangular grid (covering the combustion layer region) and "stencils" in Fig. C.2. The grid has uniform mesh widths ($\Delta x = 1/M$) and heights ($\Delta y = y_\infty/N$). Mesh points are thus located at (x_i, y_i) points with

$$\begin{aligned} x_i &= (i - 1)(\Delta x) , \qquad \text{for } 1 \le i \le M + 1 , \\ y_i &= (j - 1)(\Delta y) , \qquad \text{for } 1 \le j \le N + 1 , \end{aligned} \tag{C.8}$$

and dependent variables evaluated at those points are given corresponding (i, j) suffices. For example, then, $x_1 = 0$ (and not one), $y_{N+1} = y_\infty$, $u_{1,N+1} = u_0(0, y_\infty)$. The accurate approximation of $(u_{i,j};\ v_{i,j};\ P_{i,j};\ T_{i,j};\ \rho_{i,j})_{1\le i\le M+1,\,1\le j\le N+1}$ constitutes a numerical solution of the problem posed by Eqs. (C.1) and (C.2) together with (C.3) or (C.4). The stencils in Fig. C.2, explained below, are indicative of the types of differencing used in the codes.

Along the $y = 0$ line (where $j = 1$), $v_0 \equiv 0$. As shown in Fig. C.1, a guess at $\hat{P}_1$ along this line [actually, the $\{P_{i1}\}_{i=1}^{M+1}$ values] is also made prior to the calling of the MSWEEP subroutine. As noted in the first large block of Fig. 6 (p. 35), Eqs. (C.1) and (C.2) then reduce along this line to the following (ordinary differential equation) boundary value problem for u_0 and T_0 alone (with $\rho_0 = 1/T_0$):

$$\left(\frac{u_0}{T_0}\right)\frac{\partial u_0}{\partial x} = -\frac{\partial \hat{P}_1}{\partial x} + M_{11}\frac{\partial^2 u_0}{\partial x^2} ;$$

$$\left(\frac{u_0}{T_0}\right)\frac{\partial T_0}{\partial x} = q(x, 0) - \sigma(T_0^4 - 1) + K_1\frac{\partial^2 T_0}{\partial x^2} ;$$

$$u_0 = \frac{\partial T_0}{\partial x} = 0 \quad \text{on} \quad x = 0 ;$$

$T_0 = 1$ and either

$$\frac{\partial}{\partial x}\left(\frac{u_0}{T_0}\right) = \hat{\phi}(y) + \beta_0 u_0$$

$$\text{or } u_0 = \hat{g}(y) \qquad \text{(C.9)}$$

$$\text{on } x = 1 \ .$$

The standard finite-difference solution for this problem involves the solution of

$$\vec{F}(\vec{x}) = \vec{0} \ , \qquad \text{(C.10)}$$

where

$$\vec{x} \equiv \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{x}_{2M-3} \\ \tilde{x}_{2M-2} \\ \tilde{x}_{2M-1} \end{pmatrix} = \begin{pmatrix} u_{2,1} \\ T_{2,1} \\ u_{3,1} \\ T_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ u_{M,1} \\ T_{M,1} \\ u_{M+1,1} \end{pmatrix} \qquad \text{(C.11)}$$

and the components of $\vec{F}$ are the finite difference equations obtained from the discretization of Eq. (C.9). The Crank-Nicolson solution of Eq. (C.1)--subject to the solution of (C.9) [for given $\hat{P}_1$ (x, 0)] and the x = 0 and x = 1 (only) boundary conditions in Eqs. (C.2) together with (C.3) or (C.4)--also involves the solution of Eq. (C.10) for successive j, j = 2, 3, 4, ..., N, N + 1. For this, the components of $\vec{F}$ are slightly different from those of j = 1; and for each fixed $j(\geq 2)$, $\vec{x}$ is redefined [cf. Eq. (C.11)] as

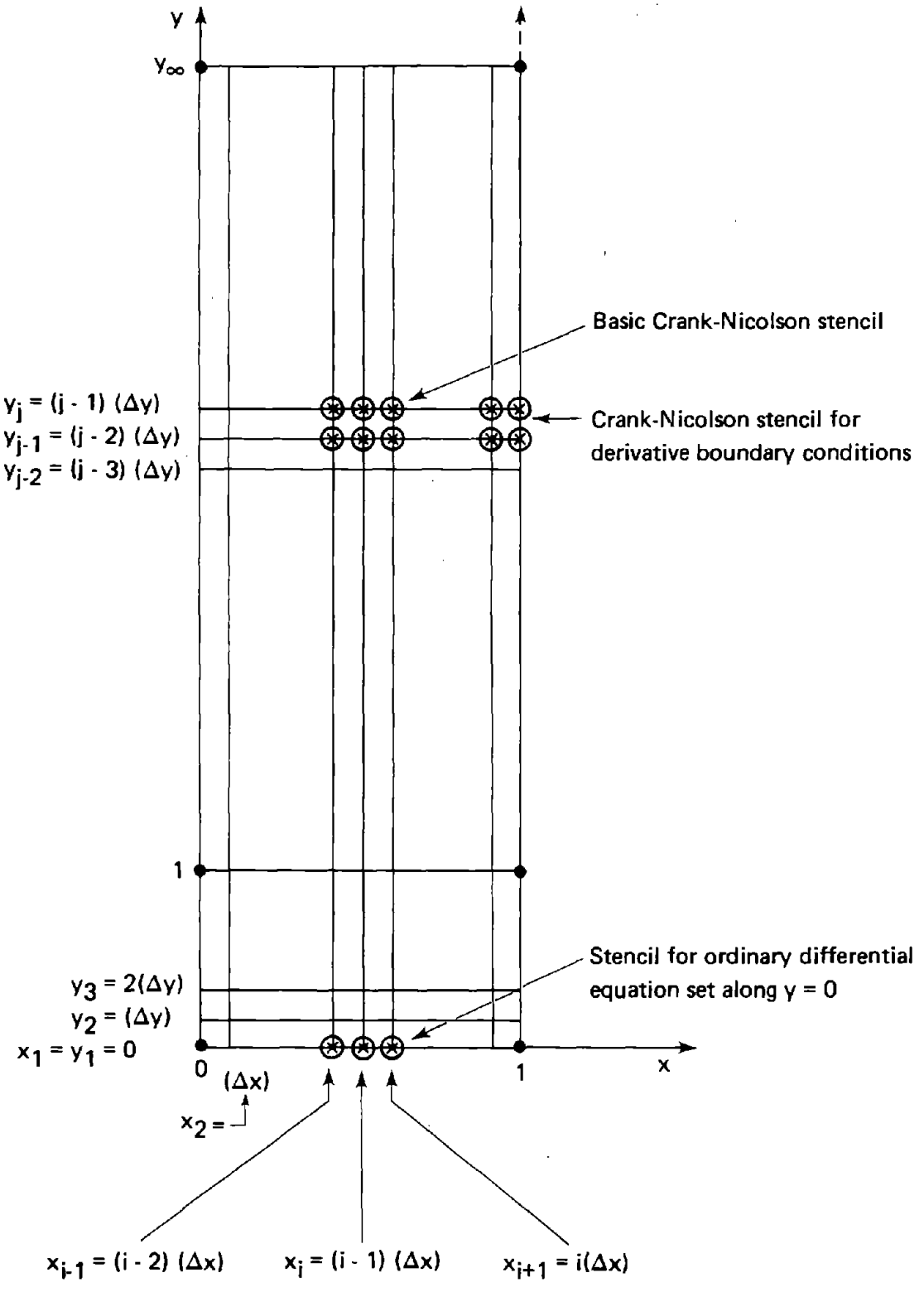

Fig. C.2--Finite difference grid and stencils for numerical solution of leading-order combustion-layer problem

$$\vec{x} \equiv \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \\ \vdots \\ \tilde{x}_{2M-3} \\ \tilde{x}_{2M-2} \\ \tilde{x}_{2M-1} \end{pmatrix} = \begin{pmatrix} u_{2,j} \\ T_{2,j} \\ u_{3,j} \\ T_{3,j} \\ \vdots \\ u_{M,j} \\ T_{M,j} \\ u_{M+1,j} \end{pmatrix} . \qquad \text{(C.12)}$$

The overall flow in subroutine MSWEEP is thus as outlined in Fig. C.3, Newton's method (Fig. C.4) being used in all cases to solve the equations represented by Eq. (C.10) because they are nonlinear.

To fully describe subroutine MSWEEP, we must define the following processes:

1. The specific finite-difference reduction of Eqs. (C.9) and (C.2) through (C.10).
2. The corresponding specifications of $J \left(= \frac{\partial \vec{F}}{\partial \vec{x}}\right)$.
3. The specific Gaussian elimination scheme used to compute $\vec{\delta}$ (as $-J^{-1}\vec{F}$).

For j = 1 (i.e., y = 0), the usual discretization for second-order ordinary differential equations is used to reduce Eq. (C.9) to Eq. (C.10). That is, for any dependent variable w, interior derivatives are replaced by the centered difference approximations

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_{i,1} \rightarrow \frac{w_{i+1,1} - 2w_{i,1} + w_{i-1,1}}{(\Delta x)^2} ;$$

$$\left(\frac{\partial w}{\partial x}\right)_{i,1} \rightarrow \frac{w_{i+1,1} - w_{i-1,1}}{2(\Delta x)} ; \qquad \text{(C.13)}$$

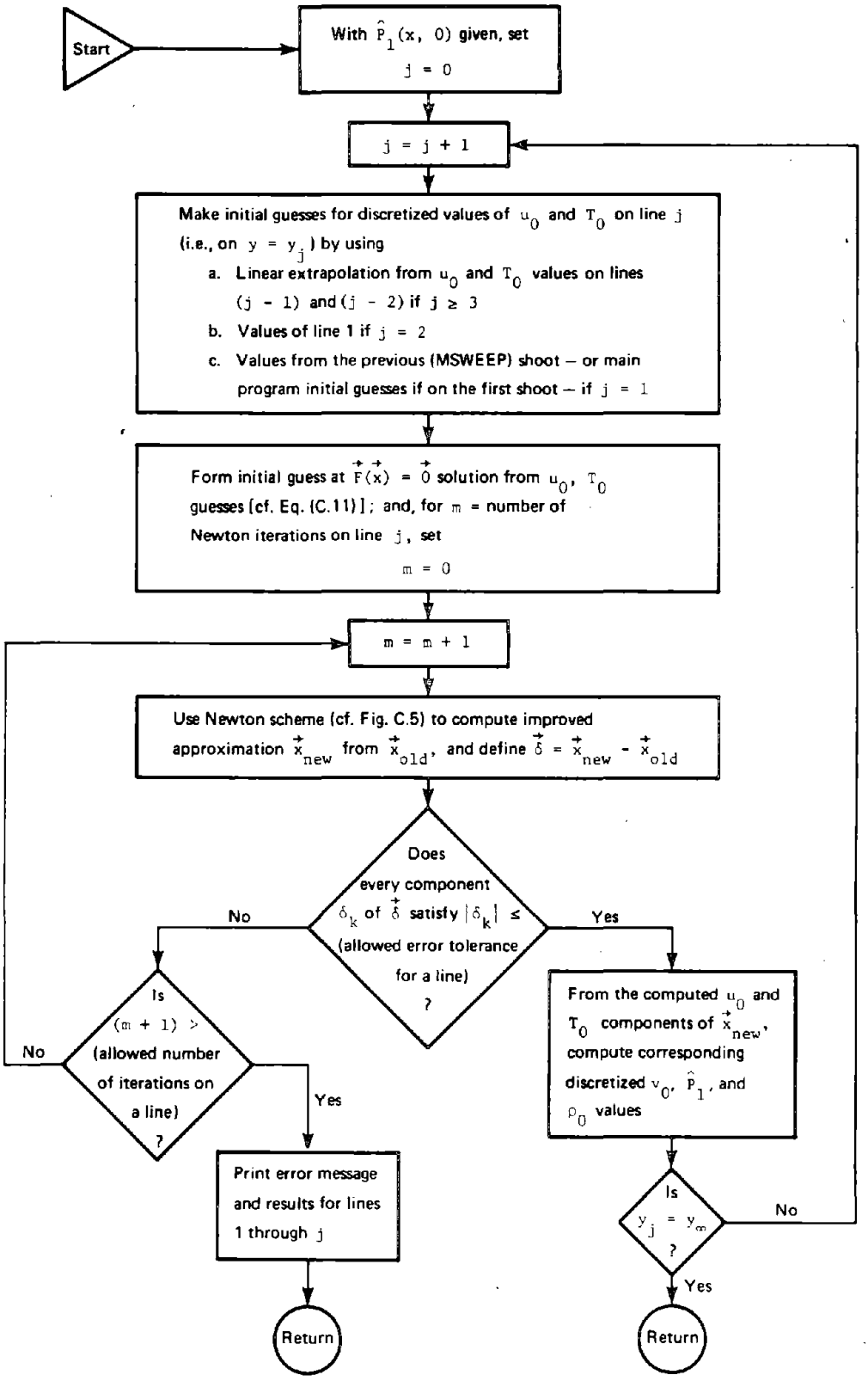

Fig. C.3--Macro flowchart of subroutine MSWEEP

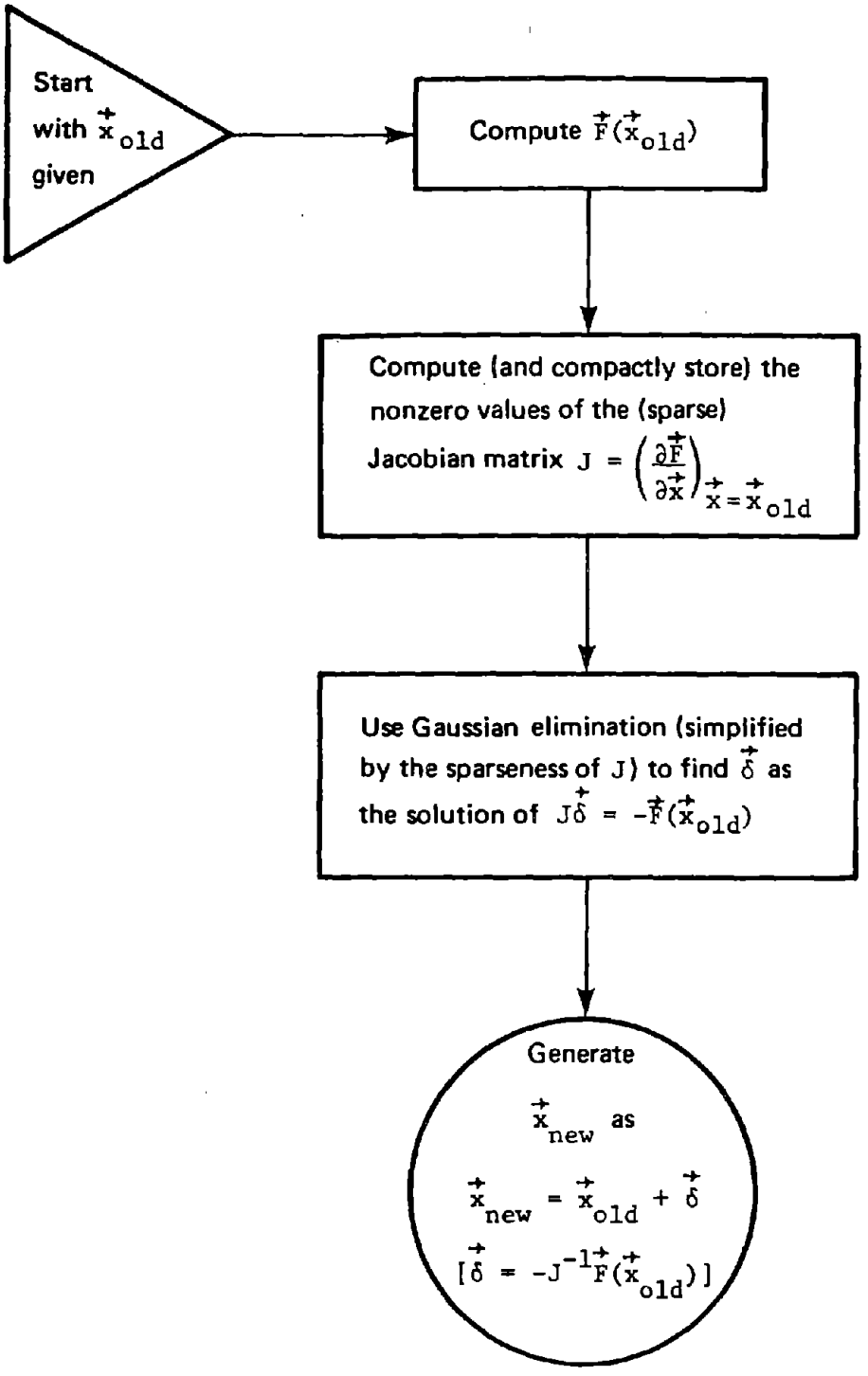

Fig. C.4--Flow diagram of basic Newton step used in subroutine MSWEEP

and boundary derivatives are taken as, for example,

$$\left(\frac{\partial w}{\partial x}\right)_{1,1} \to \frac{w_{2,1} - w_{1,1}}{(\Delta x)} . \tag{C.14}$$

As indicated by the corresponding discretization stencil at the bottom of Fig. C.2, no more than three values of u_0 and T_0 are thus involved in each component equation in Eq. (C.10) [cf. Eq. (C.11)]--and these values are at successive x_i. For each $j \geq 2$, the Crank-Nicolson scheme to be defined by Eqs. (C.15) and (C.16) is used to reduce Eqs. (C.1) and (C.2) together with (C.3) or (C.4), to Eq. (C.10). Centered differences are also used, with interior and boundary approximation stencils as shown in Fig. C.2, interior approximations being

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_{i,j} \to \frac{1}{2}\left[\frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{(\Delta x)^2} + \frac{w_{i+1,j-1} - 2w_{i,j-1} + w_{i-1,j-1}}{(\Delta x)^2}\right];$$

$$\left(\frac{\partial w}{\partial x}\right)_{i,j} \to \frac{1}{2}\left[\frac{w_{i+1,j} - w_{i-1,j}}{2(\Delta x)} + \frac{w_{i+1,j-1} - w_{i-1,j-1}}{2(\Delta x)}\right];$$

$$\left(\frac{\partial w}{\partial y}\right)_{i,j} \to \left(\frac{w_{i,j} - w_{i,j-1}}{\Delta y}\right);$$

$$(w)_{i,j} \to \frac{1}{2}\left(w_{i,j} + w_{i,j-1}\right); \tag{C.15}$$

and the boundary formulas being of the form

$$\left(\frac{\partial w}{\partial x}\right)_{1,j} \to \frac{1}{2}\left[\frac{w_{2,j} - w_{1,j}}{(\Delta x)} + \frac{w_{2,j-1} - w_{1,j-1}}{(\Delta x)}\right];$$

$$(w)_{1,j} \to \frac{1}{2}\left(w_{1,j} + w_{1,j-1}\right). \tag{C.16}$$

As in linear problems [Isaacson and Keller, 1966], the scheme has proven to be convergent, stable, and accurate for both nonlinear formulations of the combustion-layer boundary-value problem.

In effecting the reduction of Eqs. (C.9), and (C.1) and (C.2) together with (C.3) or (C.4), to Eq. (C.10) by means of Eqs. (C.13) through (C.16), it proves convenient--both notationally and computationally--to begin by decomposing $\vec{x}$ (for all j) into

$$\vec{u} = \begin{pmatrix} u_{2,j} \\ u_{3,j} \\ \cdot \\ \cdot \\ \cdot \\ u_{M,j} \\ - - - - \\ u_{M+1,j} \end{pmatrix} ; \qquad \vec{T} = \begin{pmatrix} T_{2,j} \\ T_{3,j} \\ \cdot \\ \cdot \\ \cdot \\ T_{M,j} \end{pmatrix} \tag{C.17}$$

and arranging the corresponding $\vec{F}(\vec{x})$ as

$$\vec{F} = \begin{pmatrix} G_2 \\ H_2 \\ G_3 \\ H_3 \\ \cdot \\ \cdot \\ \cdot \\ G_M \\ H_M \\ - - - - \\ G_{M+1} \end{pmatrix} , \tag{C.18}$$

where (for all relevant i) G_i and H_i represent the components that arise (as is shown below) basically from the discretization of the horizontal momentum and energy equations, respectively, about the

grid point (x_i, y_j). As mentioned above, centered differencing results in each G_i and H_i involving at most six of the various $(2M - 2)$ $u_{i,j}$ and $T_{i,j}$. Hence, the Jacobian matrix derivatives of

$$\vec{G}(\vec{u}, \vec{T}) = \begin{pmatrix} G_2 \\ G_3 \\ \vdots \\ G_M \\ \hline G_{M+1} \end{pmatrix} \quad ; \quad \vec{H}(\vec{u}, \vec{T}) = \begin{pmatrix} H_2 \\ H_3 \\ \vdots \\ H_M \end{pmatrix} \qquad (C.19)$$

with respect to $\vec{u}$ and $\vec{T}$ have the following sparse, banded forms:

$$\frac{\partial \vec{G}}{\partial \vec{u}} = \left(\begin{array}{cccccc:cc} \alpha_{11} & \alpha_{12} & & & & & 0 & 0 \\ \alpha_{20} & \alpha_{21} & \alpha_{22} & & & & & \\ & \alpha_{30} & \alpha_{31} & \alpha_{32} & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & \alpha_{M-2,0} & \alpha_{M-2,1} & \alpha_{M-2,2} & & \\ 0 & & & & \alpha_{M-1,0} & \alpha_{M-1,1} & \alpha_{M-1,2} & \\ \hline 0 & & & & & \alpha_{M,0} & \alpha_{M,1} & \end{array}\right) ;$$

$$\frac{\partial \vec{G}}{\partial \vec{T}} = \begin{pmatrix} \beta_{11} & \beta_{12} & & & & & 0 \\ \beta_{20} & \beta_{21} & \beta_{22} & & & & \\ & \beta_{30} & \beta_{31} & \beta_{32} & & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \beta_{M-2,0} & \beta_{M-2,1} & \beta_{M-2,2} \\ & & & & & \beta_{M-1,0} & \beta_{M-1,1} \\ 0 & & & & & & \\ - & - & - & - & - & - & - \\ 0 & & & & & & \beta_{M,0} \end{pmatrix} ;$$

$$\frac{\partial \vec{H}}{\partial \vec{u}} = \left(\begin{array}{ccccccc} \gamma_{11} & \gamma_{12} & & & & 0 & 0 \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & & & & \\ & \gamma_{30} & \gamma_{31} & \gamma_{32} & & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \gamma_{M-2,0} & \gamma_{M-2,1} & \gamma_{M-2,2} \\ & & & & & \gamma_{M-1,0} & \gamma_{M-1,1} \\ 0 & & & & & & \end{array} \;\middle|\; \begin{array}{c} 0 \\ \\ \\ \\ \\ \gamma_{M-1,2} \end{array}\right) ;$$

$$\begin{bmatrix} \delta_{11} & \delta_{12} & & & & & & \\ \delta_{20} & \delta_{21} & \delta_{22} & & & & & \\ & \delta_{30} & \delta_{31} & \delta_{32} & & & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & \delta_{M-2,0} & \delta_{M-2,1} & \delta_{M-2,2} \\ & & & & & & \delta_{M-1,0} & \delta_{M-1,1} \end{bmatrix} . \qquad (C.20)$$

From Eqs. (C.12) and (C.17) through (C.20), the $J = \partial\vec{F}/\partial\vec{x}$ Jacobian is then easily computed from the nonzero elements in these [Eq. (C.20)] Jacobians as

$$J = \frac{\partial \vec{F}}{\partial \vec{x}} = \left[\begin{array}{cccccccccccc|cc} \alpha_{11} & \beta_{11} & \alpha_{12} & \beta_{12} & & & & & & & & & 0 & 0 \\ \gamma_{11} & \delta_{11} & \gamma_{12} & \delta_{12} & 0 & & & & & & & & & \\ \alpha_{20} & \beta_{20} & \alpha_{21} & \beta_{21} & \alpha_{22} & \beta_{22} & & & & & & & & \\ \gamma_{20} & \delta_{20} & \gamma_{21} & \delta_{21} & \gamma_{22} & \delta_{22} & 0 & & & & & & & \\ & 0 & \alpha_{30} & \beta_{30} & \alpha_{31} & \beta_{31} & \alpha_{32} & \beta_{32} & & & & & & \\ & & \gamma_{30} & \delta_{30} & \gamma_{31} & \delta_{31} & \gamma_{32} & \delta_{32} & & & & & & \\ & & & & & & & & \ddots & \ddots & \ddots & \ddots & & \\ & & & & & & & & & \alpha_{M-2,0} & \beta_{M-2,0} & \alpha_{M-2,1} & \beta_{M-2,1} & \alpha_{M-2,2} & \beta_{M-2,2} & \\ & & & & & & & & & \gamma_{M-2,0} & \delta_{M-2,0} & \gamma_{M-2,1} & \delta_{M-2,1} & \gamma_{M-2,2} & \delta_{M-2,2} & 0 \\ & & & & & & & & & & 0 & \alpha_{M-1,0} & \beta_{M-1,0} & \alpha_{M-1,1} & \beta_{M-1,1} & \alpha_{M-1,2} \\ 0 & & & & & & & & & & & \gamma_{M-1,0} & \delta_{M-1,0} & \gamma_{M-1,1} & \delta_{M-1,1} & \gamma_{M-1,2} \\ \hline 0 & & & & & & & & & & & & 0 & \alpha_{M,0} & \beta_{M,0} & \alpha_{M,1} \end{array}\right] \qquad (C.21)$$

DIFFERENCE EQUATIONS AND JACOBIAN ELEMENTS

Specific forms for $\vec{G}$ and $\vec{H}$, and hence for $\vec{F}$ and the various α, β, γ, and δ coefficients in Eqs. (C.20) and (C.21), are derived as follows. Subject to Eq. (C.13), the first two lines of Eq. (C.9) are approximated at $x = x_i$, $2 \le i \le M$, by

$$\left(\frac{u_{i,1}}{T_{i,1}}\right)\left[\frac{u_{i+1,1} - u_{i-1,1}}{2(\Delta x)}\right] = -\left[\frac{P_{i+1,1} - P_{i-1,1}}{2(\Delta x)}\right] + M_{11}\left[\frac{u_{i+1,1} - 2u_{i,1} + u_{i-1,1}}{(\Delta x)^2}\right],$$

$$\left(\frac{u_{i,1}}{T_{i,1}}\right)\left[\frac{T_{i+1,1} - T_{i-1,1}}{2(\Delta x)}\right] = q(x_i, 0) - \sigma(T_{i,1}^4 - 1) + K_1\left[\frac{T_{i+1,1} - 2T_{i,1} + T_{i-1,1}}{(\Delta x)^2}\right] . \quad (C.22)$$

From the third and fourth lines in Eq. (C.9) and from Eq. (C.14), we then also have

$$u_{1,1} = 0 \; ; \qquad T_{1,1} = T_{2,1} \; ; \qquad T_{M+1,1} = 1 \; ; \quad (C.23)$$

and

$$\left(\frac{1}{\Delta x}\right)\left(\frac{u_{M+1,1}}{T_{M+1,1}} - \frac{u_{M,1}}{T_{M,1}}\right) = \hat{\phi}(0) + \beta_0 u_{M+1,1}$$

or

$$u_{M+1,1} = \hat{g}(0) \; . \quad (C.24)$$

For $j = 1$, we thus have for both codes

$$G_2 = \left(\frac{M_{11}}{\Delta x}\right)(u_{3,1} - 2u_{2,1}) - \left(\frac{1}{2}\right)\left(\frac{u_{2,1}}{T_{2,1}}\right)u_{3,1} - \left(\frac{1}{2}\right)(P_{3,1} - P_{1,1}) ;$$

$$H_2 = \left(\frac{K_1}{\Delta x}\right)(T_{3,1} - T_{2,1}) - \left(\frac{1}{2}\right)\left(\frac{u_{2,1}}{T_{2,1}}\right)(T_{3,1} - T_{2,1})$$

$$+ \left[q(x_2, 0) - \sigma(T_{2,1}^4 - 1)\right](\Delta x) ; \qquad (C.25)$$

and, for $3 \le i \le M$,

$$G_i = \left(\frac{M_{11}}{\Delta x}\right)(u_{i+1,1} - 2u_{i,1} + u_{i-1,1}) - \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right)(u_{i+1,1} - u_{i-1,1})$$

$$- \left(\frac{1}{2}\right)(P_{i+1,1} - P_{i-1,1}) ;$$

$$H_i = \left(\frac{K_1}{\Delta x}\right)(T_{i+1,1} - 2T_{i,1} + T_{i-1,1}) - \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right)(T_{i+1,1} - T_{i-1,1})$$

$$+ \left[q(x_i, 0) - \sigma(T_{i,1}^4 - 1)\right](\Delta x) . \qquad (C.26)$$

In this case, we also have [from Eq. (C.24)]

$$G_{M+1} = \left(\frac{1}{\Delta x}\right)\left[u_{M+1,1} - \left(\frac{u_{M,1}}{T_{M,1}}\right)\right] - \beta_0 u_{M+1,1} - \phi(0) , \qquad (C.27)$$

but G_{M+1} is not defined when $u_0 = \hat{g}(y)$ is prescribed on $x = 1$.

From Eqs. (C.25) and (C.26), the following Jacobian elements of Eq. (C.20) are then found to be the same for both codes for $j = 1$:

$$\alpha_{11} = -2\left(\frac{M_{11}}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{3,1}}{T_{2,1}}\right) ,$$

$$\alpha_{12} = \left(\frac{M_{11}}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{2,1}}{T_{2,1}}\right) ;$$

$$\beta_{11} = \left(\frac{1}{2}\right)\left(\frac{u_{2,1}u_{3,1}}{T_{2,1}^2}\right) ,$$

$$\beta_{12} = 0 ;$$

$$\gamma_{11} = -\left(\frac{1}{2}\right)\left(\frac{T_{3,1}}{T_{2,1}} - 1\right) ,$$

$$\gamma_{12} = 0 ;$$

$$\delta_{11} = -\left(\frac{K_1}{\Delta x}\right) + \left(\frac{1}{2}\right)\left(\frac{u_{2,1}T_{3,1}}{T_{2,1}^2}\right) - 4\sigma T_{2,1}^3 \, (\Delta x) ,$$

$$\delta_{12} = \left(\frac{K_1}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{2,1}}{T_{2,1}}\right) ; \qquad \text{(C.28)}$$

and, for $3 \le i \le M$,

$$\alpha_{i-1,0} = \left(\frac{M_{11}}{\Delta x}\right) + \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right) ,$$

$$\alpha_{i-1,1} = -2\left(\frac{M_{11}}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{i+1,1} - u_{i-1,1}}{T_{i,1}}\right) ,$$

$$\alpha_{i-1,2} = \left(\frac{M_{11}}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right) ;$$

$$\beta_{i-1,0} = 0 ,$$

$$\beta_{i-1,1} = \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}^2}\right)(u_{i+1,1} - u_{i-1,1}) ,$$

$$\beta_{i-1,2} = 0 ;$$

$$\gamma_{i-1,0} = 0 \ ,$$

$$\gamma_{i-1,1} = -\left(\frac{1}{2}\right)\left(\frac{T_{i+1,1} - T_{i-1,1}}{T_{i,1}}\right) \ ,$$

$$\gamma_{i-1,2} = 0 \ ;$$

$$\delta_{i-1,0} = \left(\frac{K_1}{\Delta x}\right) + \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right) \ ,$$

$$\delta_{i-1,1} = -2\left(\frac{K_1}{\Delta x}\right) + \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}^2}\right)(T_{i+1,1} - T_{i-1,1})$$

$$-4\sigma T_{i,1}^3 \ (\Delta x) \ ,$$

$$\delta_{i-1,2} = \left(\frac{K_1}{\Delta x}\right) - \left(\frac{1}{2}\right)\left(\frac{u_{i,1}}{T_{i,1}}\right) \ ; \qquad \text{(C.29)}$$

except that $\beta_{M-1,2}$ and $\delta_{M-1,2}$ are undefined if Eq. (C.3) is specified and $\alpha_{M-1,2}$, $\beta_{M-1,2}$, $\gamma_{M-1,2}$, and $\delta_{M-1,2}$ are all undefined if Eq. (C.4) is used. If Eq. (C.3) is specified, the remaining Jacobian elements are [from Eq. (C.27)]

$$\alpha_{M,0} = -\left(\frac{1}{\Delta x}\right)\left(\frac{1}{T_{M,1}}\right) \ ,$$

$$\alpha_{M,1} = \left(\frac{1}{\Delta x}\right) - \beta_0 \ ,$$

$$\beta_{M,0} = \left(\frac{1}{\Delta x}\right)\left(\frac{u_{M,1}}{T_{M,1}^2}\right) \ . \qquad \text{(C.30)}$$

For $j \geq 2$, the derivation of relevant forms for $\vec{G}$ and $\vec{H}$ begins with a preliminary rewriting of Eq. (C.1) in the following, equivalent "conservation" form:

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 \ ;$$

$$\frac{\partial}{\partial x}(\rho_0 u_0^2 + \hat{P}_1) + \frac{\partial}{\partial y}(\rho_0 u_0 v_0) = M_{11}\frac{\partial^2 u_0}{\partial x^2} \ ;$$

$$\frac{\partial \hat{P}_1}{\partial y} + \hat{A}\rho_0 = 0 \ ;$$

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \equiv \frac{\partial}{\partial x}(\rho_0 u_0 T_0) + \frac{\partial}{\partial y}(\rho_0 v_0 T_0) = K_{11}\frac{\partial^2 T_0}{\partial x^2} + \left[q(x, y) - \sigma(T_0^4 - 1)\right] \ ;$$

$$\rho_0 T_0 = 1 \ . \qquad \text{(C.31)}$$

Subject to Eq. (C.15), this set is approximated at each interior grid point (x_i, y_j), $2 \le i \le M$, of line j, $2 \le j \le N+1$, by the finite difference system

$$\left(\frac{1}{4\Delta x}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} + \left(\frac{u}{T}\right)_{i+1,j-1} - \left(\frac{u}{T}\right)_{i-1,j} - \left(\frac{u}{T}\right)_{i-1,j-1}\right] + \left(\frac{1}{\Delta y}\right)\left[\left(\frac{v}{T}\right)_{i,j} - \left(\frac{v}{T}\right)_{i,j-1}\right] = 0 \ ;$$

$$\left(\frac{1}{4\Delta x}\right)\left[\left(\frac{u^2}{T} + P\right)_{i+1,j} + \left(\frac{u^2}{T} + P\right)_{i+1,j-1} - \left(\frac{u^2}{T} + P\right)_{i-1,j} - \left(\frac{u^2}{T} + P\right)_{i-1,j-1}\right] + \left(\frac{1}{\Delta y}\right)\left[\left(\frac{uv}{T}\right)_{i,j} - \left(\frac{uv}{T}\right)_{i,j-1}\right] = \left[\frac{M_{11}}{2(\Delta x)^2}\right] \times \left(u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + u_{i+1,j-1} - 2u_{i,j-1} + u_{i-1,j-1}\right) \ ;$$

$$\left(\frac{1}{\Delta y}\right)\left(P_{i,j} - P_{i,j-1}\right) + \left(\frac{\hat{A}}{2}\right)\left[\left(\frac{1}{T}\right)_{i,j} + \left(\frac{1}{T}\right)_{i,j-1}\right] \ ;$$

$$\left(\frac{1}{4\Delta x}\right)\left(u_{i+1,j} + u_{i+1,j-1} - u_{i-1,j} - u_{i-1,j-1}\right) + \left(\frac{1}{\Delta y}\right)\left(v_{i,j} - v_{i,j-1}\right)$$

$$= \left[\frac{K_{11}}{2(\Delta x)^2}\right]\left(T_{i+1,j} - 2T_{i,j} + T_{i-1,j} + T_{i+1,j-1} - 2T_{i,j-1} + T_{i-1,j-1}\right)$$

$$+ \left(\frac{1}{2}\right)\left[q(x_i, y_j) + q(x_i, y_{j-1})\right] + \sigma\left[\left(\frac{T_{i,j} + T_{i,j-1}}{2}\right)^4 - 1\right] ; \qquad (C.32)$$

ρ_0 having been eliminated for simplicity. This system is further simplified by solving the first and third equations [in Eq. (C.32)] for $v_{i,j}$ and $P_{i,j}$ in terms of the various u and T on line j and variables on line j - 1 as

$$v_{i,j} = \left(\frac{v}{T}\right)_{i,j-1} T_{i,j} - \left(\frac{1}{4}\right)\left(\frac{\Delta y}{\Delta x}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} + \left(\frac{u}{T}\right)_{i+1,j-1}\right.$$

$$\left. - \left(\frac{u}{T}\right)_{i-1,j} - \left(\frac{u}{T}\right)_{i-1,j-1}\right] T_{i,j} ;$$

$$P_{i,j} = P_{i,j-1} - \left(\frac{\hat{A}}{2}\right)\left[\left(\frac{1}{T}\right)_{i,j} + \left(\frac{1}{T}\right)_{i,j-1}\right](\Delta y) ; \qquad (C.33)$$

and substituting these results into the second and fourth lines to generate (for $2 \le i \le M$)

$$G_i \equiv \left[\frac{M_{11}}{2(\Delta x)}\right]\left(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right) - \left(\frac{1}{4}\right)\left\{\left(\frac{u^2}{T}\right)_{i+1,j}\right.$$

$$\left. - \left(\frac{u^2}{T}\right)_{i-1,j} - \hat{A}\left(\frac{\Delta y}{2}\right)\left[\left(\frac{1}{T_{i+1,j}}\right) - \left(\frac{1}{T_{i-1,j}}\right)\right]\right\} - \left(\frac{\Delta x}{\Delta y}\right) u_{i,j}$$

$$\times \left\{\left(\frac{v}{T}\right)_{i,j-1} - \left(\frac{\Delta y}{4\Delta x}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} - \left(\frac{u}{T}\right)_{i-1,j} + \left(\frac{u}{T}\right)_{i+1,j-1}\right.\right.$$

$$\left.\left. - \left(\frac{u}{T}\right)_{i-1,j-1}\right]\right\} - R_i = 0 ;$$

$$H_i \equiv \left[\frac{K_1}{2(\Delta x)}\right]\left(T_{i+1,j} - 2T_{i,j} + T_{i-1,j}\right) - \left(\frac{1}{4}\right)\left(u_{i+1,j} - u_{i-1,j}\right)$$

$$- \left(\frac{\Delta x}{\Delta y}\right) T_{i,j}\left\{\left(\frac{v}{T}\right)_{i,j-1} - \left(\frac{\Delta y}{4\Delta x}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} - \left(\frac{u}{T}\right)_{i-1,j}\right.\right.$$

$$\left.\left. + \left(\frac{u}{T}\right)_{i+1,j-1} - \left(\frac{u}{T}\right)_{i-1,j-1}\right]\right\} + \sigma\left[\left(\frac{T_{i,j} + T_{i,j-1}}{2}\right)^4 - 1\right](\Delta x)$$

$$- \Pi_i = 0 \; ; \qquad \text{(C.34)}$$

where

$$R_i = -\left[\left[\frac{M_{11}}{2(\Delta x)}\right]\left(u_{i+1,j-1} - 2u_{i,j-1} + u_{i-1,j-1}\right) - \left(\frac{1}{4}\right)\left\{\left(\frac{u^2}{T}\right)_{i+1,j-1} - \left(\frac{u^2}{T}\right)_{i-1,j-1}\right.\right.$$

$$\left. + 2\left(P_{i+1,j-1} - P_{i-1,j-1}\right) - \hat{A}\left(\frac{\Delta y}{2}\right)\left[\left(\frac{1}{T_{i+1,j-1}}\right) + \left(\frac{1}{T_{i-1,j-1}}\right)\right]\right\}$$

$$\left. + \left(\frac{\Delta x}{\Delta y}\right)\left[\left(\frac{uv}{T}\right)_{i,j-1}\right]\right] \; ;$$

$$\Pi_i = -\left\{\left[\frac{K_1}{2(\Delta x)}\right]\left(T_{i+1,j-1} - 2T_{i,j-1} + T_{i-1,j-1}\right) - \left(\frac{1}{4}\right)\left(u_{i+1,j-1}\right.\right.$$

$$\left.\left. - u_{i-1,j-1}\right) + \left(\frac{\Delta x}{\Delta y}\right) v_{i,j-1} + \left(\frac{1}{2}\right)\left[q(x_i, y_j) + q(x_i, y_{j-1})\right](\Delta x)\right\} .$$

$$\text{(C.35)}$$

As outlined in Fig. C.3, a recursive Crank-Nicolson scheme is employed in subroutine MSWEEP to generate a numerical solution for Eq. (C.1) subject to a given $\hat{P}_1(x, 0)$. For $j = 2, 3, \ldots, N + 1$, the solution on line j is found using solution data on line $(j - 1)$, the solution just described for $j = 1$ being used to start the recursion. For each $j \geq 2$ in both codes, the relevant forms of G_i and H_i (as components of $\vec{F}$), $2 \leq i \leq M$, are therefore exactly those in Eq. (C.34) [cf. Eqs. (C.25) and (C.26)], variables with $j - 1$ suffices being known. Also known are the discretized boundary conditions

$$u_{1,j} = 0 \; ; \qquad T_{1,j} = T_{2,j} \; ; \qquad T_{M+1,j} = 1 \; ; \tag{C.36}$$

and

$$u_{M+1,j} = \hat{g}(y_j) \; , \qquad \text{if } u_0 = \hat{g}(y) \text{ is given on } x = 1 \tag{C.37}$$

[cf. Eqs. (C.23) and (C.24)]. With specification of the boundary condition in Eq. (C.3), Eq. (C.37) is replaced [using Eq. (C.16) in Eq. (C.3)] by

$$G_{M+1} \equiv \left[\frac{1}{2(\Delta x)}\right]\left[u_{M+1,j} - \left(\frac{u}{T}\right)_{M,j} + u_{M+1,j-1} - \left(\frac{u}{T}\right)_{M,j-1}\right]$$

$$- \left(\frac{1}{2}\right)\left[\beta_0\left(u_{M+1,j} + u_{M+1,j-1}\right) + \hat{\phi}(y_j) + \hat{\phi}(y_{j-1})\right] = 0 \; . \tag{C.38}$$

This defines the last component of $\vec{G}$.

In both codes, the following Jacobian elements are then found [from Eq. (C.34)] to be the same for each fixed choice of $j \geq 2$:

$$\alpha_{11} = -2\left[\frac{M_{11}}{2(\Delta x)}\right] - \left\{\left(\frac{\Delta x}{\Delta y}\right)\left(\frac{v}{T}\right)_{2,j-1} - \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)_{3,j} + \left(\frac{u}{T}\right)_{3,j-1}\right]\right\} ,$$

$$\alpha_{12} = \left[\frac{M_{11}}{2(\Delta x)}\right] - \left(\frac{1}{4}\right)\left[2\left(\frac{u}{T}\right)_{3,j} - \left(\frac{u_{2,j}}{T_{3,j}}\right)\right] \; ;$$

$$\beta_{11} = \left(\frac{1}{4}\right)\left[\hat{A}\left(\frac{\Delta y}{2}\right)\left(\frac{1}{T_{2,j}}\right)^2\right] \; ,$$

$$\beta_{12} = \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)^2_{3,j} - \hat{A}\left(\frac{\Delta y}{2}\right)\left(\frac{1}{T_{3,j}}\right)^2 - u_{2,j}\left(\frac{u}{T^2}\right)_{3,j}\right]$$

since $T_{1,j} = T_{2,j}$ [cf. Eq. (C.36)];

$$\gamma_{11} = 0 \ ,$$

$$\gamma_{12} = -\left(\frac{1}{4}\right)\left(1 - \frac{T_{2,j}}{T_{3,j}}\right) \ ;$$

$$\delta_{11} = -\left[\frac{K_1}{2(\Delta x)}\right] - \left\{\left(\frac{\Delta x}{\Delta y}\right)\left(\frac{v}{T}\right)_{2,j-1} - \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)_{3,j} + \left(\frac{u}{T}\right)_{3,j-1}\right]\right\}$$

$$+ 2\sigma\left(\frac{T_{2,j} + T_{2,j-1}}{2}\right)^3 (\Delta x) \ ,$$

$$\delta_{12} = \left[\frac{K_1}{2(\Delta x)}\right] - \left(\frac{1}{4}\right)\left[T_{2,j}\left(\frac{u}{T^2}\right)_{3,j}\right] \ ; \qquad \text{(C.39)}$$

and, for $3 \le i \le M$,

$$\alpha_{i-1,0} = \left[\frac{M_{11}}{2(\Delta x)}\right] + \left(\frac{1}{4}\right)\left[2\left(\frac{u}{T}\right)_{i-1,j} - \left(\frac{u_{i,j}}{T_{i-1,j}}\right)\right] \ ,$$

$$\alpha_{i-1,1} = -2\left[\frac{M_{11}}{2(\Delta x)}\right] - \left\{\left(\frac{\Delta x}{\Delta y}\right)\left(\frac{v}{T}\right)_{i,j-1} - \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} + \left(\frac{u}{T}\right)_{i+1,j-1}\right.\right.$$

$$\left.\left. - \left(\frac{u}{T}\right)_{i-1,j} - \left(\frac{u}{T}\right)_{i-1,j-1}\right]\right\} \ ,$$

$$\alpha_{i-1,2} = \left[\frac{M_{11}}{2(\Delta x)}\right] - \left(\frac{1}{4}\right)\left[2\left(\frac{u}{T}\right)_{i+1,j} - \left(\frac{u_{i,j}}{T_{i+1,j}}\right)\right] \ ;$$

$$\beta_{i-1,0} = -\left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)^2_{i-1,j} - \hat{A}\left(\frac{\Delta y}{2}\right)\left(\frac{1}{T_{i-1,j}}\right)^2 - u_{i,j}\left(\frac{u}{T^2}\right)_{i-1,j}\right] \ ,$$

$$\beta_{i-1,1} = 0 \ ,$$

$$\beta_{i-1,2} = \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)^2_{i+1,j} - \hat{A}\left(\frac{\Delta y}{2}\right)\left(\frac{1}{T_{i+1,j}}\right)^2 - u_{i,j}\left(\frac{u}{T^2}\right)_{i+i,j}\right] \ ;$$

$$\gamma_{i-1,0} = \left(\frac{1}{4}\right)\left[1 - \left(\frac{T_{i,j}}{T_{i-1,j}}\right)\right] ,$$

$$\gamma_{i-1,1} = 0 ,$$

$$\gamma_{i-1,2} = -\left(\frac{1}{4}\right)\left[1 - \left(\frac{T_{i,j}}{T_{i+1,j}}\right)\right] ;$$

$$\delta_{i-1,0} = \left[\frac{K_1}{2(\Delta x)}\right] + \left(\frac{1}{4}\right)\left[T_{i,j}\left(\frac{u}{T^2}\right)_{i-1,j}\right] ,$$

$$\delta_{i-1,1} = -2\left[\frac{K_1}{2(\Delta x)}\right] - \left\{\left(\frac{\Delta x}{\Delta y}\right)\left(\frac{v}{T}\right)_{i,j-1} - \left(\frac{1}{4}\right)\left[\left(\frac{u}{T}\right)_{i+1,j} + \left(\frac{u}{T}\right)_{i+1,j-1} - \left(\frac{u}{T}\right)_{i-1,j} - \left(\frac{u}{T}\right)_{i-1,j-1}\right]\right\} + 2\sigma\left(\frac{T_{i,j} + T_{i,j-1}}{2}\right)^3 (\Delta x) ,$$

$$\delta_{i-1,2} = \left[\frac{K_1}{2(\Delta x)}\right] - \left(\frac{1}{4}\right)\left[T_{i,j}\left(\frac{u}{T^2}\right)_{i+1,j}\right] . \qquad \text{(C.40)}$$

However, we do not define $\beta_{M-1,2}$ and $\delta_{M-1,2}$ in both versions or $\alpha_{M-1,2}$ and $\gamma_{M-1,2}$ in the version where $u_0(1, y)$ is given. In the general version (FIRE), which uses Eq. (C.3), the remaining Jacobian elements are [from Eq. (C.38)];

$$\alpha_{M,0} = -\left[\frac{1}{2(\Delta x)}\right]\left(\frac{1}{T_{M,j}}\right) .$$

$$\alpha_{M,1} = \left(\frac{1}{2}\right)\left[\frac{1}{(\Delta x)} - \beta_0\right] ;$$

$$\beta_{M,0} = \left[\frac{1}{2(\Delta x)}\right]\left(\frac{u_{M,j}}{T^2_{M,j}}\right) . \qquad \text{(C.41)}$$

The detailed specification of $\vec{G}$, $\vec{H}$, and J (= $\partial\vec{F}/\partial\vec{x}$) is now complete for all cases. $J\vec{\delta} = -\vec{F}$ is solved for $\vec{\delta}$ using (cf. Fig. C.4) Gaussian elimination [Isaacson and Keller, 1966]. For efficiency, standard sparse-matrix methods are employed: only nonzero elements of J [cf. Eq. (C.21)] are stored, and operations involving zero elements are automatically omitted.

PROGRAM LISTING

FORTRAN listings of both codes are now presented. Table C.2 is the FIRE code listing; Table C.3 is the FIRE2 code listing.

Table C.2

LISTING OF FIRE CODE

```
FORTRAN-VIID:R04e00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUME0TATIT0A1PA0RA08:004-00$099.0/     1 V
  1            $BATCH                                                                                                  1
  2            C                                                                                                       2
  3            C                                                                                                       3
  4  000000I         PROGRAM FIRE                                                                                      4
  5            C                                                                                                       5
  6            C                                                                                                       6
  7  000006I         COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                                     7
  8                1       X(41),Y(241),                                                                              8
  9                2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                                               9
 10                3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                                 10
 11                4       ETLINE,MAXITL,ETOLP,MAXITP,                                                                11
 12                5       QVAL,XNU,XK,QPRIM,                                                                         12
 13                6       ALPHA,BETA,BLOSS,IGUESS,                                                                   13
 14                7       IPRINT,ITTY,KOUT                                                                           14
 15           C                                                                                                       15
 16           C                                                                                                       16
 17           C     READ AND WRITE OUT INPUTS                                                                         17
 18           C                                                                                                       18
 19           C                                                                                                       19
 20  000006I         ITTY=1                                                                                            20
 21  00000EI         IPRINT=2                                                                                          21
 22                                                                                                                    22
 23  000016I         READ (ITTY,1)  A, SIGMA, XNUIN, XKIN                                                              23
 24  000040I         READ (ITTY,3)  YMAX, M, N                                                                         24
 25  000068I         READ (ITTY,2)  ETLINE, ETOLP, MAXITL, MAXITP                                                      25
 26  000094I         READ (ITTY,4)  ALPHA, BETA, BLOSS, IGUESS                                                         26
 27  0000C0I    1    FORMAT (4F10.0)                                                                                   27
 28  0000CEI    2    FORMAT (2F10.0, 2I10)                                                                             28
 29  0000E0I    3    FORMAT (F10.0, 2I10)                                                                              29
 30  0000F0I    4    FORMAT (3F10.0, I10)                                                                              30
 31                                                                                                                    31
 32  000100I         DX=1./FLOAT(M)                                                                                    32
 33  000126I         DY=YMAX/FLOAT(N)                                                                                  33
 34                                                                                                                    34
 35  00014AI         WRITE(IPRINT,10)                                                                                  35
 36  000164I   10    FORMAT (//// 2X 'FIRE: SOLUTION OF COMBUSTION LAYER ',                                            36
 37                1  'BOUNDARY VALUE PROBLEM' ///)                                                                   37
 38  0001B2I         WRITE(IPRINT,12)A,SIGMA,XNUIN,XKIN                                                                38
 39  0001DCI   12    FORMAT('  SYSTEM PARAMETERS:   A =' F8.3 '  SIGMA =' F8.3                                        39
 40                1  '  NU =' F8.3 '  K =' F8.3 )                                                                    40
 41  000226I         WRITE(IPRINT,14)M,DX,N,DY                                                                         41
 42  000250I   14    FORMAT(/ '  MESH:   M =' I5 '  DX =' F8.3 '  N =' I5                                            42
 43                1        '   DY =' F8.3)                                                                         43
 44  00028AI         WRITE(IPRINT,16)ETLINE,MAXITL,ETOLP,MAXITP                                                        44
 45  0002B4I   16    FORMAT(/ '  ITERATION PARAMETERS:   ETLINE = ' F8.6                                               45
 46                1       '  MAXITL =' I5 '  ETOLP = ' F8.6 '   MAXITP =' I5)                                    46
 47  0003I2I         WRITE (IPRINT,18)  ALPHA, BETA, BLOSS                                                             47
 48  000338I   18    FORMAT (/ '  INLET PARAMETERS:   ALPHA =' F8.3 '  BETA ='                                          48
 49                1  F8.3 '  BLOSS =' F8.3)                                                                         49
 50           C                                                                                                       50
 51           C                                                                                                       51
 52           C     INITIALIZATION                                                                                    52
 53           C                                                                                                       53
 54           C                                                                                                       54
 55  00037EI         DYDX=DY/DX                                                                                        55
 56  000390I         DXDY=DX/DY                                                                                        56
 57  0003A2I         MP=M+1                                                                                            57
```

Table C.2--Continued

```
FORTRAN-VIID:R04.00USED RESTRICTED RIGHTS AS STATED IN LICENSE L-0335        ***, SEE DOCUMENTATION]PAGE A00.004-R04.00 99.9/      2 V
 58  0003B0I          NP=N+1                                                                                          58
 59                                                                                                                   59
 60  0003BEI          X(1)=0.0                                                                                        60
 61  0003CAI          Y(1)=0.0                                                                                        61
 62  0003D6I          DO 30 I=2,M                                                                                     62
 63  0003EAI          X(I)=DX*(FLOAT(I)-1.)                                                                           63
 64  00041AI    30    CONTINUE                                                                                        64
 65  000432I          DO 32 J=2,N                                                                                     65
 66  000446I          Y(J)=DY*(FLOAT(J)-1.)                                                                           66
 67  000476I    32    CONTINUE                                                                                        67
 68  00048EI          X(MP)=1.0                                                                                       68
 69  0004A2I          Y(NP)=YMAX                                                                                      69
 70                                                                                                                   70
 71  0004B6I          DO 35 I=1,MP                                                                                    71
 72  0004CAI          V(I,1)=0.0                                                                                      72
 73  0004DEI    35    CONTINUE                                                                                        73
 74                                                                                                                   74
 75  0004F6I          DO 40 J=1,NP                                                                                    75
 76  00050AI          U(1,J)=0.0                                                                                      76
 77  000524I          T(MP,J)=1.0                                                                                     77
 78  000544I          RHO(MP,J)=1.0                                                                                   78
 79  000564I    40    CONTINUE                                                                                        79
 80               C                                                                                                  80
 81               C    MAKE INITIAL GUESSES ALONG Y = 0.0                                                            81
 82               C                                                                                                  82
 83  00057CI          IZERO=0                                                                                         83
 84  000584I          KK=1                                                                                            84
 85  00058CI          CALL BCFUNC(KK,IZERO,IZERO)                                                                     85
 86  0005ACI          KOUT = 0                                                                                        86
 87               C                                                                                                  87
 88               C                                                                                                  88
 89               C                                                                                                  89
 90               C    OVERALL ITERATION: MARCHING FROM Y=0.0 TO                                                     90
 91               C                       Y=YMAX,RECURSIVELY FINDING                                                 91
 92               C                       SOLN. ON Y=Y(J+1) FROM SOLN.                                               92
 93               C                       ON Y=Y(J), AND REPEATING UNTIL                                             93
 94               C                       PX=0 ON Y=YMAX                                                             94
 95               C                                                                                                  95
 96               C                                                                                                  96
 97                                                                                                                   97
 98                                                                                                                   98
 99  0005B4I          NNN=0                                                                                           99
100  00058CI    50    CONTINUE                                                                                       100
101  0005BCI          NNN=NNN+1                                                                                      101
102  0005CAI          IF(NNN.GE.MAXITP) GO TO 950                                                                    102
103                                                                                                                  103
104                                                                                                                  104
105  0005E2I          CALL MSWEEP                                                                                    105
106  0005EAI          IF (KOUT .EQ. 1)  GO TO 980                                                                    106
107               C                                                                                                  107
108               C    OVERALL CONVERGENCE CHECK PLUS                                                                108
109               C    NEWTON MODIFICATIONS                                                                          109
110               C                                                                                                  110
111  000600I          DO 110 I=1,M                                                                                   111
112  000614I          IP=I+1                                                                                         112
113  000622I          IF(ABS(P(I,NP)-P(IP,NP)).GE.(DX*FTOLP)) GO TO 115                                              113
114  00068EI    110   CONTINUE                                                                                       114
```

Table C.2--Continued

```
FORTRAN-VIID:RO$ERNSED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUMENTATIT08]PAGMA08$004-00$899.0/      3 V
115  0006A6I          GO TO 900                                                                                          115
116                                                                                                                      116
117  0006ACI    115   CONTINUE                                                                                           117
118  0006ACI          XOMEGA=0.9                                                                                         118
119  0006B8I          DO 120 I=1,M                                                                                       119
120  0006CCI          P(I,1)=P(I,1)+XOMEGA*(P(MP,NP)-P(I,NP))                                                            120
121  000722I    120   CONTINUE                                                                                           121
122  00073AI          GO TO 50                                                                                           122
123               C                                                                                                     123
124               C                                                                                                     124
125               C     STANDARD PRINTOUT                                                                               125
126               C                                                                                                     126
127               C                                                                                                     127
128  00073EI    900   CONTINUE                                                                                           128
129  00073EI          DO 920 J=1,NP                                                                                      129
130  000752I          CALL SPRINT(J)                                                                                     130
131  000768I    920   CONTINUE                                                                                           131
132  000780I          GO TO 980                                                                                          132
133               C                                                                                                     133
134               C                                                                                                     134
135               C     ERROR PRINTOUT                                                                                  135
136               C                                                                                                     136
137               C                                                                                                     137
138  000786I    950   CONTINUE                                                                                           138
139  000786I          WRITE(IPRINT,960) MAXITP                                                                           139
140  0007A4I    960   FORMAT(////,'  NO CONVERGENCE IN',I6,' OVERALL                                                     140
141                 1               ITERATIONS ON P',///)                                                              141
142                                                                                                                      142
143  000806I          CALL SPRINT(NP)                                                                                    143
144  000818I          STOP 0                                                                                             144
145  000840I    980   CONTINUE                                                                                           145
146                                                                                                                      146
147  000840I          WRITE (8)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                      147
148  00087CI          WRITE (8)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                      148
149  0008ACI          DO 990  J = 1, NP                                                                                  149
150  0008C0I          WRITE (8)  (U(I,J), I = 1, MP)                                                                     150
151  00094CI    990   CONTINUE                                                                                           151
152  000964I          DO 991  J = 1, NP                                                                                  152
153  000978I          WRITE (8)  (V(I,J), I = 1, MP)                                                                     153
154  000A00I    991   CONTINUE                                                                                           154
155  000A18I          WRITE (9)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                      155
156  000A54I          WRITE (9)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                      156
157  000A84I          DO 992  J = 1, NP                                                                                  157
158  000A98I          WRITE (9)  (T(I,J), I = 1, MP)                                                                     158
159  000920I    992   CONTINUE                                                                                           159
160  000B38I          DO 993  J = 1, NP                                                                                  160
161  000B4CI          WRITE (9)  (P(I,J), I = 1, MP)                                                                     161
162  000D41I    993   CONTINUE                                                                                           162
163               C     WRITE (7)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                   163
164               C     WRITE (7)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                   164
165               C     DO 994  J = 1, NP                                                                               165
166               C     WRITE (7)  (RHO(I,J), I = 1, MP)                                                                166
167               C994  CONTINUE                                                                                        167
168  000E1CI          STOP                                                                                               168
169  000E34I          END                                                                                                169

NO  ERRORS:F70  R04-00     MAINPROG    FIRE      07/17/81  10:20:04  TABLE SPACE:   5 KB
```

Table C.2--Continued

```
FORTRAN-VIID:RQ4E@@SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUME@TATI@@IPAC@@A@@:004-P@6@99.F/      5 V
  1  000000I          SUBROUTINE BCFUNC(KEY,II,JJ)                                                    170
  2                                                                                                    171
  3                                                                                                    172
  4  0000041          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                     173
  5                  1       X(41),Y(241),                                                           174
  6                  2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                            175
  7                  3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                              176
  8                  4       ETLINE,MAXITL,ETOLP,MAXITP,                                             177
  9                  5       QVAL,XNU,XK,PRIM,                                                       178
 10                  6       ALPHA,BETA,BLOSS,IGUESS,                                                179
 11                  7       IPRINT,ITTY,KOUT                                                        180
 12                                                                                                   181
 13                                                                                                   182
 14  0000041          GO TO (100, 200, 300, 400, 500)  KEY                                            183
 15            C                                                                                      184
 16            C                                                                                      185
 17            C     INITIAL GUESSES FOR Y=0.0 - CAN BE CHANGED                                       186
 18            C                                                                                      187
 19            C                                                                                      188
 20  000036I   100   GO TO (125, 150, 175)  IGUESS                                                    189
 21                                                                                                   190
 22  00005EI   125   DO 126  I = 1, MP                                                                191
 23  000072I         READ (ITTY, 127)  U(I,1), T(I,1), P(I,1)                                        192
 24  0001001   126   CONTINUE                                                                         193
 25  000118I   127   FORMAT (3F10.0)                                                                  194
 26  000126I         GO TO 900                                                                       195
 27                                                                                                   196
 28  00012CI   150   CONTINUE                                                                         197
 29  00012CI         DO 160  I = 1, MP                                                               198
 30  000140I         READ (ITTY,161)  P(I,1)                                                         199
 31  000174I   160   CONTINUE                                                                         200
 32  00018CI   161   FORMAT (F10.0)                                                                   201
 33                                                                                                   202
 34  0001981   175   CONTINUE                                                                         203
 35  0001981         DO 180  I = 1, MP                                                               204
 36  0001ACI         IF (IGUESS .EQ. 3)  P(I,1) = 0.                                                 205
 37  0001D0I         T(I,1) = 1.76-0.76*X(I)                                                         206
 38  0001FAI         CC = 1.6                                                                        207
 39  0002061         CCC=1.9                                                                         208
 40  000212I         EP = EXP(CC*X(I))                                                               209
 41  00024AI         EM = EXP(-CC*X(I))                                                              210
 42  00028AI         U(I,1) = -CCC*(EP-EM)/(EP+EM)                                                   211
 43  0002C0I    180  CONTINUE                                                                        212
 44  0002D8I         GO TO 900                                                                       213
 45            C                                                                                      214
 46            C                                                                                      215
 47            C     Q(X,Y) PRESCRIPTION - CAN BE CHANGED                                             216
 48            C                                                                                      217
 49            C                                                                                      218
 50  0002DEI   200   CONTINUE                                                                         219
 51  0002DEI         IF(JJ.EQ.1)TAVG=T(II,JJ)                                                        220
 52  00030EI         JJM=JJ-1                                                                        221
 53  00031CI         IF(JJ.GE.2)TAVG=0.5*(T(II,JJ)+T(II,JJM))                                        222
 54                                                                                                   223
 55  00036CI         QVAL=1.0-SIGMA*(TAVG**4-1.0)                                                    224
 56                                                                                                   225
 57  0003941         IF(Y(JJ).GT.1.0) QVAL=QVAL-1.0                                                  226
```

Table C.2--Continued

```
FORTRAN-VIID:R04&E00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUME07&II081PAC0&AQB:004-R0$R99.2/      6 V
 58              C                                                                                    227
 59              C                                                                                    228
 60              C      NU(X,Y) PRESCRIPTION - CAN BE CHANGED                                         229
 61              C                                                                                    230
 62              C                                                                                    231
 63  0003C0I     300    CONTINUE                                                                      232
 64  0003C0I            XNU = XNUIN                                                                   233
 65  0003CCI            CNU=0.0                                                                       234
 66  0003D8I            XNU=XNU*EXP(-CNU*X(II))                                                       235
 67  00041CI            IF(X(II).GT.1.0)XNU=0.0                                                       236
 68  000442I            GO TO 900                                                                     237
 69              C                                                                                    238
 70              C                                                                                    239
 71              C      K(X,Y) PRESCRIPTION - CAN BE CHANGED                                          240
 72              C                                                                                    241
 73              C                                                                                    242
 74  000448I     400    CONTINUE                                                                      243
 75  000448I            XK = XKIN                                                                     244
 76  000454I            CK=0.0                                                                        245
 77  000460I            XK=XK*EXP(-CK*X(II))                                                          246
 78  0004A4I            IF(X(II).GT.1.0)XK=0.0                                                        247
 79  0004CAI            GO TO 900                                                                     248
 80              C                                                                                    249
 81              C      QPRIME COMPUTATION - CAN BE CHANGED                                           250
 82              C                                                                                    251
 83  0004D0I     500    CONTINUE                                                                      252
 84  0004D0I            IF(JJ.EQ.1)TAVG=T(II,JJ)                                                      253
 85  0005D0I            JJM=JJ-1                                                                      254
 86  00050EI            IF(JJ.GE.2)TAVG=(T(II,JJ)+T(II,JJM))/2.                                       255
 87                                                                                                   256
 88  00055EI            QPRIM=-4.*SIGMA*(TAVG**3)                                                     257
 89                                                                                                   258
 90  000588I     900    CONTINUE                                                                      259
 91  000588I            RETURN                                                                        260
 92  00058EI            END                                                                           261

NO  ERRORS:F7D  R04-00     SUBROUTINE  BCFUNC    07/17/81  10:20:09  TABLE SPACE:   4 KB
STATEMENT BUFFER:   20 LINES/1321 BYTES    STACK SPACE: 174 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
```

Table C.2--Continued

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FORTRAN-VIID:RQ4E00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATI0N1PAC8A00:004-0@5899.8/      7 V
  1  000000I          SUBROUTINE SPRINT(JJ)                                                                    262
  2                                                                                                            263
  3                                                                                                            264
  4  000004I          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                              265
  5                  1       X(41),Y(241),                                                                    266
  6                  2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                                     267
  7                  3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                       268
  8                  4       ETLINF,MAXITL,ETOLP,MAXITP,                                                      269
  9                  5       QVAL,XNU,XK,QPRIM,                                                               270
 10                  6       ALPHA,BETA,BLOSS,IGUESS,                                                         271
 11                  7       IPRINT,ITTY,KOUT                                                                 272
 12                                                                                                           273
 13  000004I          DIMENSION  PX(41), VISC(41), DIFF(41), HEAT(41)                                          274
 14            C                                                                                              275
 15            C                                                                                              276
 16            C     COMPUTE PX, UXX, TXX AND Q(T) TERMS                                                      277
 17            C                                                                                              278
 18            C                                                                                              279
 19  000004I          DO 50  I = 1, MP                                                                         280
 20  0002B0I          IP = I + 1                                                                               281
 21  0002BEI          IM = I - 1                                                                               282
 22  0002CCI          DO 10  KK = 2, 4                                                                         283
 23  0002D4I          CALL BCFUNC (KK, I, JJ)                                                                  284
 24  0002F4I     10   CONTINUE                                                                                 285
 25  00030AI          IF (I .EQ. 1)  GO TO 20                                                                  286
 26  000320I          IF (I .EQ. MP)  GO TO 30                                                                 287
 27                                                                                                           288
 28  000338I          PX(I) = (P(IP,JJ)-P(IM,JJ))/(2.*DX)                                                     289
 29  000386I          VISC(I) = (U(IP,JJ)-2.*U(I,JJ)+U(IM,JJ))*XNU/(DX*DX)                                    290
 30  0003FCI          DIFF(I) = (T(IP,JJ)-2.*T(I,JJ)+T(IM,JJ))*XK/(DX*DX)                                     291
 31  000472I          HEAT(I) = QVAL                                                                           292
 32  000484I          GO TO 50                                                                                 293
 33                                                                                                           294
 34  00048AI     20   PX(I) = 0.                                                                               295
 35  00049CI          VISC(I) = 0.                                                                             296
 36  0004AEI          DIFF(I) = 2.*(T(IP,JJ)-T(I,JJ))*XK/(DX*DX)                                              297
 37  000508I          HEAT(I) = QVAL                                                                           298
 38  00051AI          GO TO 50                                                                                 299
 39                                                                                                           300
 40  000520I     30   PX(I) = (P(I,JJ)-P(IM,JJ))/DX                                                           301
 41  000566I          VISC(I) = VISC(IM)                                                                       302
 42  00057EI          DIFF(I) = DIFF(IM)                                                                       303
 43  000596I          HEAT(I) = QVAL                                                                           304
 44                                                                                                           305
 45  0005A8I     50   CONTINUE                                                                                 306
 46            C                                                                                              307
 47            C                                                                                              308
 48            C     STANDARD PRINTOUT                                                                        309
 49            C                                                                                              310
 50            C                                                                                              311
 51  0005C0I          WRITE(IPRINT,100)JJ,Y(JJ)                                                                312
 52  0005F8I     100  FORMAT(//// ' FOR J =' I5 '   (Y =' F8.3 '):')                                           313
 53  000620I          WRITE(IPRINT,120)                                                                        314
 54  00063CI     120  FORMAT(//,'          I     X        U        V        T',                               315
 55                 1    '       RHO      P' 21X 'PX      NU*UXX    K*TXX    Q(T)' /)                       316
 56  0006A2I          DO 150 I=1,MP                                                                            317
 57  0006B6I          WRITE(IPRINT,140)I,X(I),U(I,JJ),V(I,JJ),T(I,JJ),RHO(I,JJ),                               318
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Table C.2--Continued

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FORTRAN-VIID:R04680SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATION1PAGE#408:004-00599.0/      8 V
 58                    1              P(I,JJ), PX(I), VISC(I), DIFF(I), HEAT(I)                          319
 59  000804I    140    FORMAT(3X,I6,6F8.3, 15X, 4F8.3)                                                    320
 60  00081CI    160    CONTINUE                                                                           321
 61  000834I           RETURN                                                                             322
 62  000838I           END                                                                                323

NO  ERRORS:F7D  R04-00      SUBROUTINE  SPRINT     07/17/81  10:20:12  TABLE SPACE:    3 KB
STATEMENT BUFFER:    20 LINES/1321 BYTES     STACK SPACE: 171 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
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Table C.2--Continued

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FORTRAN-VIID:RO$E00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATI?0$IPA(0400$004-P0$899.?/     9 V
  1  0000001          SUBROUTINE MSWEEP                                                                          324
  2                                                                                                              325
  3                                                                                                              326
  4  0000041          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                                327
  5                  1       X(41),Y(241),                                                                      328
  6                  2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                                       329
  7                  3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                         330
  8                  4       ETLINE,MAXITL,ETOLP,MAXITP,                                                        331
  9                  5       QVAL,XNU,XK,QPRIM,                                                                 332
 10                  6       ALPHA,BETA,BLOSS,IGUESS,                                                           333
 11                  7       IPRINT,ITTY,KOUT                                                                   334
 12                                                                                                              335
 13  0000041          DIMENSION XX(200),FF(200),AZ(100),A1(100),                                                 336
 14                  1         A2(100),BZ(100),B1(100),B2(100),                                                 337
 15                  2         CZ(100),C1(100),C2(100),DZ(100),                                                 338
 16                  3         D1(100),D2(100),XMAT(7,200),                                                     339
 17                  4         DNEW(200)                                                                        340
 18                                                                                                              341
 19                                                                                                              342
 20  0000041          MM=M-1                                                                                     343
 21  00321AI          MB=2*(M-1)+1                                                                               344
 22  0032301          MBM = MB - 1                                                                               345
 23  00323EI          MBM2 = MB - 2                                                                              346
 24  00324CI          MBM3 = MB - 3                                                                              347
 25                                                                                                              348
 26  00325AI          DO 800 J=1,NP                                                                              349
 27  00326EI          JM=J-1                                                                                     350
 28  00327CI          JM2=J-2                                                                                    351
 29             C                                                                                               352
 30             C                                                                                               353
 31             C     INITIAL GUESSES FOR U AND T ON LINE J                                                     354
 32             C                                                                                               355
 33             C                                                                                               356
 34  00328AI          DO 20 I=2,MP                                                                               357
 35  00329EI          IU=2*(I-1)-1                                                                               358
 36  0032841          IT=2*(I-1)                                                                                 359
 37  0032C81          IF(J.EQ.1) GO TO 14                                                                        360
 38  0032DEI          IF(J .EQ. 2) GO TO 12                                                                      361
 39             C                                                                                               362
 40             C     LINEAR EXTRAPOLATION USED FOR J .GE. 3                                                    363
 41             C                                                                                               364
 42  0032F41          U(I,J)=2.*U(I,JM)-U(I,JM2)                                                                 365
 43  0033481          T(I,J)=2.*T(I,JM)-T(I,JM2)                                                                 366
 44  00339CI          GO TO 14                                                                                   367
 45             C                                                                                               368
 46             C     PREVIOUS VALUES (=INITIAL GUESSES FOR NNN=1) USED FOR J = 1 OR J = 2                      369
 47             C                                                                                               370
 48  0033A2I    12    U(I,J)=U(I,JM)                                                                             371
 49  0033D61          T(I,J)=T(I,JM)                                                                             372
 50  00340AI    14    CONTINUE                                                                                   373
 51  00340AI          XX(IU)=U(I,J)                                                                              374
 52  0034301          XX(IT)=T(I,J)                                                                              375
 53  0034561          IF(I.EQ.2)T(1,J)=T(I,J)                                                                    376
 54  0034941    20    CONTINUE                                                                                   377
 55             C                                                                                               378
 56             C                                                                                               379
 57             C                                                                                               380
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Table C.2--Continued

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FORTRAN-V11D:RD$6@@SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336      ***, SEE DOCUME@7&II@81PA(@&A@8:004-P@$899.@/     10 V
 58              C     OVERALL VECTOR NEWTON ITERATION                                                    381
 59              C     USING GAUSS ELIMINATION ON LINEAR EQNS.                                            382
 60              C                                                                                        383
 61              C                                                                                        384
 62              C                                                                                        385
 63  0034ACI           NUMITL = 0                                                                         386
 64  0034B4I   100     NUMITL=NUMITL+1                                                                    387
 65  0034C2I           IF(NUMITL.GT.MAXITL)GO TO 900                                                      388
 66                                                                                                       389
 67                                                                                                       390
 68  0034DAI           DO 200 I=2,MP                                                                      391
 69  0034EEI           IM=I-1                                                                             392
 70  0034FCI           IP=I+1                                                                             393
 71  00350AI           IU=2*(I-1)-1                                                                       394
 72  003520I           IT=2*(I-1)                                                                         395
 73  003534I           IF(J.EQ.1) JM=1                                                                    396
 74                                                                                                       397
 75  00354CI           KK=2                                                                               398
 76  003554I           CALL BCFUNC(KK,I,J)                                                                399
 77  003574I           KK=3                                                                               400
 78  00357CI           CALL BCFUNC(KK,I,JM)                                                               401
 79  00359CI           XNUM=XNU                                                                           402
 80  0035A8I           KK=3                                                                               403
 81  0035B0I           CALL BCFUNC(KK,I,J)                                                                404
 82  0035D0I           KK=4                                                                               405
 83  0035D8I           CALL BCFUNC(KK,I,JM)                                                               406
 84  0035F8I           XKM=XK                                                                             407
 85  003604I           KK=4                                                                               408
 86  00360CI           CALL BCFUNC(KK,I,J)                                                                409
 87  00362CI           KK=5                                                                               410
 88  003634I           CALL BCFUNC(KK,I,J)                                                                411
 89                                                                                                       412
 90  003654I           IF(J.GE.2) GO TO 110                                                               413
 91              C                                                                                        414
 92              C                                                                                        415
 93              C     NEWTON SET-UP FOR J = 1                                                            416
 94              C                                                                                        417
 95              C                                                                                        418
 96  00366AI           IF(I.GE.MP) GO TO 112                                                              419
 97  003682I           RR=0.5*(P(IP,1)-P(IM,1))                                                           420
 98  0036AAI           PI=0.                                                                              421
 99                                                                                                       422
100  003686I           FF(IU)=(U(IP,1)-2.*U(I,1)+U(IM,1))*XNU/DX                                          423
101                   1       -0.5*(U(IP,1)-U(IM,1))*U(I,1)/T(I,1)-RR                                    424
102                                                                                                      425
103  003746I           FF(IT)=(T(IP,1)-2.*T(I,1)+T(IM,1))*XK/DX                                           426
104                   1       -0.5*(T(IP,1)-T(IM,1))*U(I,1)/T(I,1)+DX*QVAL                               427
105                                                                                                      428
106  0037DEI           IF(IM.GE.2) GO TO 108                                                              429
107  0037F4I           AZ(1)=0.0                                                                          430
108  0037FEI           A1(1)=-2.*XNU/DX-0.5*U(3,1)/T(2,1)                                                 431
109  003830I           A2(1)=XNU/DX-0.5*U(2,1)/T(2,1)                                                     432
110  003854I           BZ(1)=0.0                                                                          433
111  00385EI           B1(1)=0.5*U(3,1)*U(2,1)/(T(2,1)**2)                                                434
112  003884I           B2(1)=0.0                                                                          435
113  00388EI           CZ(1)=0.0                                                                          436
114  0038B8I           C1(1)=-0.5*(T(3,1)/T(2,1)-1.0)                                                     437
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Table C.2--Continued

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FORTRAN-VIID:RD4EBNSED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUMENTATI08IPACBAB8:004-P06M99.8/     II V
0038BCI          C2(1)=0.0
0038C6I          DZ(1)=0.0
0038D0I          D1(1)=-XK/DX+0.5*T(3,1)*U(2,1)/(T(2,1)**2)
                1        +DX*QPRIM
00391AI          D2(1)=XK/DX-0.5*U(2,1)/T(2,1)
00393EI          GO TO 200

003944I    108   AZ(IM)=XNU/DX+0.5*U(I,1)/T(I,1)
003980I          A1(IM)=-2.*XNU/DX-0.5*(U(IP,1)-U(IM,1))/T(I,1)
0039D8I          A2(IM)=XNU/DX-0.5*U(I,1)/T(I,1)
003A14I          BZ(IM)=0.0
003A26I          B1(IM)=0.5*(U(IP,1)-U(IM,1))*U(I,1)/(T(I,1)**2)
003A86I          B2(IM)=0.0
003A98I          CZ(IM)=0.0
003AAAI          C1(IM)=-0.5*(T(IP,1)-T(IM,1))/T(I,1)
003AEEI          C2(IM)=0.0
003B00I          DZ(IM)=XK/DX+0.5*U(I,1)/T(I,1)
003B3CI          D1(IM)=-2.*XK/DX+0.5*(T(IP,1)-T(IM,1))*U(I,1)
                1        /(T(I,1)**2)+DX*QPRIM
003BC6I          D2(IM)=XK/DX-0.5*U(I,1)/T(I,1)

003C02I          IF(I.LT.M) GO TO 200

003C1AI          D2(IM)=0.0

003C2CI          GO TO 200
           C
           C
           C     NEWTON SET-UP FOR J .GE. 2
           C
           C
003C32I    110   CONTINUE
003C32I          IF(I.GE.MP) GO TO 112
003C4AI          RR=(U(IP,JM)-2.*U(I,JM)+U(IM,JM))*XNUM
                1   /(2.*DX)-((U(IP,JM)**2)/T(IP,JM)
                1   -(U(IM,JM)**2)/T(IM,JM)+2.*(P(IP,JM)
                3   -P(IM,JM))-0.5*A*DY*(1./T(IP,JM)
                4   -1./T(IM,JM)))/4.
003DDCI          RR=-RR-U(I,JM)*V(I,JM)/T(I,JM)*DXDY
003E44I          PI=-((T(IP,JM)-2.*T(I,JM)+T(IM,JM))*XKM
                1   /(2.*DX)-(U(IP,JM)-U(IM,JM))/4.+V(I,JM)*DXDY)

003F1AI          FF(IU)=(U(IP,J)-2.*U(I,J)+U(IM,J))*XNU
                1        /(2.*DX)-0.25*((U(IP,J)**2)/T(IP,J)
                2        -(U(IM,J)**2)/T(IM,J)-0.5*A*DY
                3        *(1./T(IP,J)-1./T(IM,J)))-U(I,J)*
                4        (DXDY*V(I,JM)/T(I,JM)-0.25*(U(IP,J)
                5        /T(IP,J)+U(IP,JM)/T(IP,JM)-U(IM,J)
                6        /T(IM,J)-U(IM,JM)/T(IM,JM)))-RR

0041BCI          FF(IT)=(T(IP,J)-2.*T(I,J)+T(IM,J))*XK
                1        /(2.*DX)-0.25*(U(IP,J)-U(IM,J))
                2        -T(I,J)*(DXDY*V(I,JM)/T(I,JM)-0.25
                3        *(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)
                4        -U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))
                5        +DX*QVAL-PI
```

Table C.2--Continued

```
FORTRAN-VIID:RD4680SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***. SEE DOCUME8TAI708]PA[8A86:004-8038899.V/     12 V
172  0043881          GO TO 114                                                                                    495
173             C                                                                                                 496
174             C                                                                                                 497
175             C     PRESCRIPTION OF INLET VELOCITY BC - CAN BE CHANGED                                          498
176             C                                                                                                 499
177             C     -(DV)/(DY) = D(RHO*U)/(DX)                                                                  500
178             C                = BLOSS*U+BIN                                                                    501
179             C                                                                                                 502
180             C     NOMINAL V CASE:                                                                             503
181             C     BLOSS = 0.0, V = ALPHA*TANH((BETA/ALPHA)*Y)                                                 504
182             C                                                                                                 505
183             C                                                                                                 506
184  0043BEI    112  CONTINUE                                                                                     507
185  0043BEI         BYA = BETA*Y(J)/ALPHA                                                                       508
186  0043DEI         SECH = 2./(EXP(BYA)+EXP(-BYA))                                                              509
187  0044381          BIN = -BETA*(SECH**2)                                                                      510
188                                                                                                              511
189             C     ALTERNATIVE V CASE:                                                                         512
190             C     BLOSS=0.0, V = ALPHA*Y*EXP(-BETA*Y)                                                         513
191             C                                                                                                 514
192             C     BIN=-ALPHA*(1.0-BETA*Y(J))*EXP(-BETA*Y(J))                                                  515
193             C                                                                                                 516
194                                                                                                              517
195  00445AI         FF(IJ)=(1.0-0.5*DX*BLOSS)*U(MP,J)-U(M,J)                                                    518
196                 1       *(1.0/T(M,J)+0.5*DX*BLOSS)-BIN*DX                                                   519
197  0044FAI         IF(J.GE.2) FF(IU)=FF(IU)-((1.0/T(M,JM)+0.5*DX*BLOSS                                         520
198                 1                    )*U(M,JM)-(1.0-0.5*DX*BLOSS)*U(MP,JM))                                 521
199                 2                     -BIN*DX                                                               522
200  0045BAI         FF(IJ)=FF(IU)/DX                                                                            523
201  0045DCI         GO TO 130                                                                                   524
202                                                                                                              525
203  0045E2I    114  CONTINUE                                                                                    526
204  0045E2I         IF(IM.GE.2)GO TO 120                                                                        527
205  0045F8I         AZ(1)=0.0                                                                                   528
206  0046021         A1(1)=-2.*XNU/(2.*DX)-(DXDY*V(2,JM)/T(2,JM)                                                 529
207                 1       -0.25*(U(3,J)/T(3,J)+U(3,JM)/T(3,JM)))                                              530
208  0046B2I         A2(1)=XNU/(2.*DX)-0.25*(2.*U(3,J)-U(2,J))/T(3,J)                                            531
209  0047141         BZ(1)=0.0                                                                                   532
210  00471EI         B1(1)=0.25*(0.5*A*DY/(T(2,J)**2))                                                           533
211  0047581         B2(1)=0.25*((U(3,J)**2-U(2,J)*U(3,J))/(T(3,J)                                               534
212                 1       **2)-0.5*A*DY/(T(3,J)**2))                                                          535
213  0048041         CZ(1)=0.0                                                                                   536
214  00480EI         C1(1)=0.0                                                                                   537
215  0048181         C2(1)=-0.25*(1.-T(2,J)/T(3,J))                                                              538
216  00485AI         DZ(1)=0.0                                                                                   539
217  0048641         D1(1)=-XK/(2.*DX)-(DXDY*V(2,JM)/T(2,JM)                                                     540
218                 1       -0.25*(U(3,J)/T(3,J)+U(3,JM)/T(3,JM)))                                              541
219                 2       +0.5*DX*QPRIM                                                                       542
220  0049221         D2(1)=XK/(2.*DX)-0.25*T(2,J)*U(3,J)/(T(3,J)**2)                                             543
221  0049881         GO TO 200                                                                                   544
222                                                                                                              545
223  00498EI    120  AZ(IM)=XNU/(2.*DX)+0.25*(2.*U(IM,J)-U(I,J))/T(IM,J)                                         546
224  0049C0I         A1(IM)=-2.*XNU/(2.*DX)-(DXDY*V(I,JM)/T(I,JM)                                                547
225                 1       -0.25*(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)                                            548
226                 2       -U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))                                                549
227  0049561         A2(IM)=XNU/(2.*DX)-0.25*(2.*U(IP,J)-U(I,J))/T(IP,J)                                         550
228  0049D41         BZ(IM)=-0.25*((U(IM,J)-U(I,J))*U(IM,J)/(T(IM,J)                                             551
```

Table C.2--Continued

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FORTRAN-VIIO:R04E00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336      ***. SEE DOCUMENTATI00IPAC0A00:004-0006099.0/     13 V
229                     1       **2)-0.5*A*DY/(T(IM,J)**2))                                                552
230   004CA6I                    B1(IM)=0.0                                                                  553
231   004CBAI                    B2(IM)=0.25*((U(IP,J)-U(I,J))*U(IP,J)/(T(IP,J)                              554
232                     1       **2)-0.5*A*DY/(T(IP,J)**2))                                                555
233   004D82I                    C2(IM)=0.25*(1.-T(I,J)/T(IM,J))                                             556
234   004DD0I                    C1(IM)=0.0                                                                  557
235   004DE2I                    C2(IM)=-0.25*(1.-T(I,J)/T(IP,J))                                            558
236   004E38I                    DZ(IM)=XK/(2.*DX)+0.25*T(I,J)*U(IM,J)/(T(IM,J)**2)                          559
237   004EC4I                    D1(IM)=-2.*XK/(2.*DX)-(DXDY*V(I,JM)/T(I,JM)                                 560
238                     1       -0.25*(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)                                    561
239                     2       -U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))                                        562
240                     3       +0.5*DX*QPRIM                                                               563
241   005020I                    D2(IM)=XK/(2.*DX)-0.25*T(I,J)*U(IP,J)/(T(IP,J)**2)                          564
242                                                                                                         565
243   0050ACI                    IF(I.LT.M)GO TO 200                                                         566
244   0050C4I                    B2(IM)=0.0                                                                  567
245   0050D8I                    D2(IM)=0.0                                                                  568
246   0050EAI                    GO TO 200                                                                   569
247                                                                                                         570
248   0050F0I   130              CONTINUE                                                                    571
249   0050F0I                    AZ(IM)=-(1.0/T(M,J)+0.5*DX*BLOSS)/DX                                        572
250   005140I                    A1(IM)=(1.0-0.5*DX*BLOSS)/DX                                                573
251   00516EI                    A2(IM)=0.0                                                                  574
252   005182I                    BZ(IM)= U(M,J)/(T(M,J)**2)/DX                                               575
253   0051E0I                    B1(IM)=0.0                                                                  576
254   0051F4I                    B2(IM)=0.0                                                                  577
255   005208I                    CZ(IM)=0.0                                                                  578
256   00521CI                    C1(IM)=0.0                                                                  579
257   005230I                    C2(IM)=0.0                                                                  580
258   005242I                    DZ(IM)=0.0                                                                  581
259   005254I                    D1(IM)=0.0                                                                  582
260   005266I                    D2(IM)=0.0                                                                  583
261                                                                                                         584
262   005278I   200              CONTINUE                                                                    585
263               C                                                                                         586
264               C                                                                                         587
265               C     SPARSE (JACOBIAN) MATRIX COMPACTIFICATION                                           588
266               C                                                                                         589
267               C                                                                                         590
268   005290I                    DO 250 IM=1,M                                                               591
269   0052A4I                    IU=2*IM-1                                                                   592
270   0052B4I                    IT=2*IM                                                                     593
271   0052C2I                    XMAT(1,IU)=0.0                                                              594
272   0052DAI                    XMAT(2,IU)=AZ(IM)                                                           595
273   0052FAI                    XMAT(3,IU)=BZ(IM)                                                           596
274   00531AI                    XMAT(4,IU)=A1(IM)                                                           597
275   00533AI                    XMAT(5,IU)=B1(IM)                                                           598
276   00535AI                    XMAT(6,IU)=A2(IM)                                                           599
277   00537AI                    XMAT(7,IU)=B2(IM)                                                           600
278   00539AI                    IF(IM.GE.M) GO TO 250                                                       601
279   0053B2I                    XMAT(1,IT)=CZ(IM)                                                           602
280   0053D2I                    XMAT(2,IT)=DZ(IM)                                                           603
281   0053F0I                    XMAT(3,IT)=C1(IM)                                                           604
282   005410I                    XMAT(4,IT)=D1(IM)                                                           605
283   00542EI                    XMAT(5,IT)=C2(IM)                                                           606
284   00544EI                    XMAT(6,IT)=D2(IM)                                                           607
285   00546CI                    XMAT(7,IT)=0.0                                                              608
```

Table C.2--Continued

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FORTRAN-VIID:R04680SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATIONIPAGE0008:004-R05899.0/     14 V
286  0054841    250   CONTINUE                                                                                    609
287               C                                                                                             610
288               C                                                                                             611
289               C     GAUSS ELIMINATION TO SOLVE JD=-F                                                        612
290               C                                                                                             613
291               C                                                                                             614
292  00549CI            DO 500 I=1,MBM                                                                          615
293  0054B0I            KMAX=3                                                                                  616
294  0054B8I            IF(I.EQ.MBM2) KMAX=2                                                                    617
295  0054D2I            IF(I.EQ.MBM) KMAX=1                                                                     618
296                                                                                                             619
297  0054ECI            DO 490 K=1,KMAX                                                                         620
298  005500I            IK=I+K                                                                                  621
299  005512I            M4K=4-K                                                                                 622
300  005520I            PIVOT=XMAT(M4K,IK)/XMAT(4,I)                                                            623
301                                                                                                             624
302  005550I            DO 480 LL=4,7                                                                           625
303  005558I            ML=LL+K                                                                                 626
304  00556AI            XMAT(ML,IK)=XMAT(ML,IK)-PIVOT*XMAT(LL,I)                                                627
305  00558AI  480       CONTINUE                                                                                628
306                                                                                                             629
307  005500I            FF(IK)=FF(IK)-PIVOT*FF(I)                                                               630
308  005502I  490       CONTINUE                                                                                631
309  00561AI  500       CONTINUE                                                                                632
310                                                                                                             633
311  005632I            DNEW(MB)=-FF(MB)/XMAT(4,MB)                                                             634
312  005666I            DNEW(MBM)=-(FF(MBM)+XMAT(5,MBM)*DNEW(MB))/XMAT(4,MBM)                                   635
313  005688I            DNEW(MBM2)=-(FF(MBM2)+XMAT(5,MBM2)*DNEW(MBM)                                            636
314                   1             +XMAT(6,MBM2)*DNEW(MB))/XMAT(4,MBM2)                                      637
315                                                                                                             638
316  00572AI            DO 550 I=1,MBM3                                                                         639
317  00573EI            IB=MBM2-I                                                                               640
318  005750I            XXX=FF(IB)                                                                              641
319  005764I            DO 540 K=5,7                                                                            642
320  00576CI            IBP=IB+K-4                                                                              643
321  005780I            XXX=XXX+XMAT(K,IB)*DNEW(IBP)                                                            644
322  005798I  540       CONTINUE                                                                                645
323  0057C6I            DNEW(IB)=-XXX/XMAT(4,IB)                                                                646
324  0057F2I  550       CONTINUE                                                                                647
325               C                                                                                             648
326               C                                                                                             649
327               C     CHECK FOR NEWTON CONVERGENCE                                                            650
328               C                                                                                             651
329               C                                                                                             652
330  00580AI            IFLAG=0                                                                                 653
331  005812I            DO 420 IB=1,MB                                                                          654
332  005826I            XX(IB)=XX(IB)+DNEW(IB)                                                                  655
333  00584EI            IF(ABS(DNEW(IB)).GT.ETLINE)IFLAG=1                                                      656
334  00588CI  420       CONTINUE                                                                                657
335                                                                                                             658
336  0058A4I            DO 450 I=2,MP                                                                           659
337  0058B8I            IU=2*(I-1)-1                                                                            660
338  0058CEI            IT=2*(I-1)                                                                              661
339  0058E2I            U(I,J)=XX(IU)                                                                           662
340  00590AI            IF(I.GE.MP) GO TO 450                                                                   663
341  005922I            T(I,J)=XX(IT)                                                                           664
342  00594AI            IF(I.EQ.2)T(1,J)=T(I,J)                                                                 665
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Table C.2--Continued

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FORTRAN-VIID:R04e00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUME@TATI001PAC@A00:004-R05099.9/     15 V
343  005988I   450   CONTINUE                                                                    666
344                                                                                              667
345  0059A0I         IF(IFLAG.EQ.0)GO TO 700                                                     668
346  005986I         GO TO 100                                                                   669
347             C                                                                                670
348             C                                                                                671
349             C     SUCCESSFUL LINE SWEEP TERMINATION                                          672
350             C                                                                                673
351             C                                                                                674
352  0059BAI   700   CONTINUE                                                                    675
353                                                                                              676
354  0059BAI         DO 750 I=1,MP                                                               677
355                                                                                              678
356  0059CEI         IF (J .EQ. 1)  GO TO 740                                                    679
357                                                                                              680
358  0059E4I         IP=I+1                                                                      681
359  0059F2I         IF(I.EQ.MP)IP=I                                                             682
360  005A10I         IM=I-1                                                                      683
361  005A1EI         IF(I.EQ.1)IM=I                                                              684
362  005A3AI         FRAC=0.25                                                                   685
363  005A46I         IF(I.EQ.1)FRAC=0.5                                                          686
364  005A62I         IF(I.EQ.MP)FRAC=0.5                                                         687
365                                                                                              688
366  005A80I         V(I,J)=V(I,JM)*T(I,J)/T(I,JM)-FRAC*DYDX                                     689
367                 1       *T(I,J)*(U(IP,J)/T(IP,J)+U(IP,JM)                                    690
368                 2       /T(IP,JM)-U(IM,J)/T(IM,J)-U(IM,JM)                                   691
369                 3       /T(IM,JM))                                                           692
370                                                                                              693
371  0058E8I         P(I,J)=P(I,JM)-0.5*A*DY*(1./T(I,J)+1./T(I,JM))                              694
372  005C74I   740   RHO(I,J)=1./T(I,J)                                                          695
373                                                                                              696
374  005CAEI   750   CONTINUE                                                                    697
375  005CC6I   800   CONTINUE                                                                    698
376  005CDEI         GO TO 980                                                                   699
377             C                                                                                700
378             C                                                                                701
379             C     ERROR PRINTOUT - NO CONVERGENCE                                            702
380             C                                                                                703
381             C                                                                                704
382  005CE4I   900   CONTINUE                                                                    705
383  005CE4I         WRITE(IPRINT,905)  J, NNN, MAXITL                                           706
384  005D0CI   905   FORMAT(///,'  NO NEWTON CONVERGENCE ON LINE',                               707
385                 1       I6 ' OF OVERALL SWEEP' I6 ' IN' I6 ' ITERATIONS')                    708
386  005D64I         KOUT=1                                                                      709
387  005D6CI         DO 920 JA=1,J                                                               710
388  005D80I         CALL SPRINT(JA)                                                             711
389  005D98I   920   CONTINUE                                                                    712
390  005DB0I         STOP 1                                                                      713
391                                                                                              714
392  005DD4I   980   CONTINUE                                                                    715
393  005DD4I         RETURN                                                                      716
394  005DDCI         END                                                                         717

NO  ERRORS:F77  R04-00     SUBROUTINE  MSWEEP    07/17/81  10:20:37  TABLE SPACE:   6 KB
STATEMENT BUFFER:   20 LINES/1321 BYTES    STACK SPACE: 228 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
```

Table C.3

LISTING OF FIRE2 CODE

```
FORTRAN-VIID:RU$E@@SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUME@TATI@@IPA@@A@T:204-@05@99.@/      1 V
  1            $BATCH                                                                                              1
  2            C                                                                                                   2
  3            C                                                                                                   3
  4  000000I         PROGRAM FIRE2                                                                                 4
  5            C                                                                                                   5
  6            C                                                                                                   6
  7  000006I         COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                                   7
  8                 1       X(41),Y(241),                                                                         8
  9                 2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                                          9
 10                 3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                           10
 11                 4       ETLINE,MAXITL,ETOLP,MAXITP,                                                          11
 12                 5       QVAL,XNU,XK,QPRIM,                                                                   12
 13                 6       KOUT, IGUESS,                                                                        13
 14                 7       IPRINT,ITTY                                                                          14
 15            C                                                                                                  15
 16            C                                                                                                  16
 17            C     READ AND WRITE OUT INPUTS                                                                    17
 18            C                                                                                                  18
 19            C                                                                                                  19
 20  000006I         ITTY=1                                                                                       20
 21  00000EI         IPRINT=2                                                                                     21
 22                                                                                                               22
 23  000016I         READ (ITTY,1)  A, SIGMA, XNUIN, XKIN                                                         23
 24  000040I         READ (ITTY,1)  YMAX, ETLINE, ETOLP                                                           24
 25  000068I         READ (ITTY,2)  M, N, MAXITL, MAXITP                                                          25
 26  000094I         READ (ITTY,2)  KOUT, IGUESS                                                                  26
 27  0000B8I    1    FORMAT (4F10.0)                                                                              27
 28  0000C6I    2    FORMAT (4I10)                                                                                28
 29                                                                                                               29
 30  0000D2I         DX=1./FLOAT(M)                                                                               30
 31  0000F6I         DY=YMAX/FLOAT(N)                                                                             31
 32                                                                                                               32
 33  000114I         WRITE(IPRINT,10)                                                                             33
 34  000134I   10    FORMAT (//// 2X 'FIRE: SOLUTION OF COMBUSTION LAYER ',                                       34
 35                1  'BOUNDARY VALUE PROBLEM' ///)                                                             35
 36  000182I         WRITE(IPRINT,12)A,SIGMA,XNUIN,XKIN,YMAX                                                      36
 37  0001B0I   12    FORMAT('  SYSTEM PARAMETERS:   A =' F8.3 '  SIGMA =' F8.3                                   37
 38                1  '  NU =' F8.3 '  K =' F8.3 '   YMAX =' F8.3)                                             38
 39  000208I         WRITE(IPRINT,14)M,DX,N,DY                                                                    39
 40  000234I   14    FORMAT(/ '  MESH:   M =' I5 '  DX =' F8.3 '  N =' I5                                        40
 41                1       '   DY =' F8.3)                                                                      41
 42  00026EI         WRITE(IPRINT,16)ETLINE,MAXITL,ETOLP,MAXITP, IGUESS                                           42
 43  00029CI   16    FORMAT(/ '  ITERATION PARAMETERS:   ETLINE = ' F8.6                                          43
 44                1       '  MAXITL =' I5 '  ETOLP = ' F8.6 '   MAXITP =' I5                                44
 45                2   '   IGUESS = ' I2)                                                                       45
 46            C                                                                                                  46
 47            C                                                                                                  47
 48            C     INITIALIZATION                                                                               48
 49            C                                                                                                  49
 50            C                                                                                                  50
 51  00030AI         DYDX=DY/DX                                                                                   51
 52  00031CI         DXDY=DX/DY                                                                                   52
 53  00032EI         MP=M+1                                                                                       53
 54  00033CI         NP=N+1                                                                                       54
 55                                                                                                               55
 56  00034AI         X(1)=0.0                                                                                     56
 57  000356I         Y(1)=0.0                                                                                     57
```

Table C.3--Continued

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FORTRAN-VIID:RQ4E0BSED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUME0TATI0N1PA6BAGE1204-PR0GR99.F/      2 V
 58  000362I          DO 30 I=2,M                                                                                    58
 59  000376I          X(I)=DX*(FLOAT(I)-1.)                                                                          59
 60  0003A6I    30    CONTINUE                                                                                       60
 61  0003BEI          DO 32 J=2,N                                                                                    61
 62  0003D2I          Y(J)=DY*(FLOAT(J)-1.)                                                                          62
 63  0004021    32    CONTINUE                                                                                       63
 64  00041AI          X(MP)=1.0                                                                                      64
 65  00042EI          Y(NP)=YMAX                                                                                     65
 66                                                                                                                  66
 67  000442I          DO 35 I=1,MP                                                                                   67
 68  000456I          V(I,1)=0.0                                                                                     68
 69  00046AI    35    CONTINUE                                                                                       69
 70                                                                                                                  70
 71  000482I          DO 40 J=1,NP                                                                                   71
 72  000496I          U(1,J)=0.0                                                                                     72
 73  0004B0I          T(MP,J)=1.0                                                                                    73
 74  0004D0I          RHO(MP,J)=1.0                                                                                  74
 75  0004F0I    40    CONTINUE                                                                                       75
 76             C                                                                                                    76
 77             C     PRESCRIBE U ALONG X = 1.0 AND MAKE INITIAL GUESSES ALONG Y = 0.0                               77
 78             C                                                                                                    78
 79  000508I          IZERO=0                                                                                        79
 80  0005101          KK=1                                                                                           80
 81  000518I          CALL BCFUNC(KK,IZERO,IZERO)                                                                    81
 82             C                                                                                                    82
 83             C                                                                                                    83
 84             C                                                                                                    84
 85             C     OVERALL ITERATION: MARCHING FROM Y=0.0 TO                                                      85
 86             C                        Y=YMAX,RECURSIVELY FINDING                                                  86
 87             C                        SOLN. ON Y=Y(J+1) FROM SOLN.                                                87
 88             C                        ON Y=Y(J), AND REPEATING UNTIL                                              88
 89             C                        PX=0 ON Y=YMAX                                                              89
 90             C                                                                                                    90
 91             C                                                                                                    91
 92                                                                                                                  92
 93                                                                                                                  93
 94  000538I          NNN=0                                                                                          94
 95  000540I    50    CONTINUE                                                                                       95
 96  000540I          NNN=NNN+1                                                                                      96
 97  00054EI          IF(NNN.GE.MAXITP) GO TO 950                                                                    97
 98                                                                                                                  98
 99                                                                                                                  99
100  000566I          CALL MSWEEP                                                                                   100
101  00056EI          IF (KOUT .EQ. 0)  GO TO 980                                                                   101
102             C                                                                                                   102
103             C     CALL SPRINT(NNN)                                                                              103
104             C     IF(NNN.EQ.1) GOTO 993                                                                         104
105             C                                                                                                   105
106                                                                                                                 106
107             C                                                                                                   107
108             C     OVERALL CONVERGENCE CHECK PLUS                                                                108
109             C     NEWTON MODIFICATIONS                                                                          109
110             C                                                                                                   110
111  0005841          DO 110 I=1,M                                                                                  111
112  000598I          IP=I+1                                                                                        112
113  0005A6I          IF(ABS(P(I,NP)-P(IP,NP)).GE.(DX*ETOLP)) GO TO 115                                             113
114  0006121    110   CONTINUE                                                                                      114
```

Table C.3--Continued

```
FORTRAN-VIID:RU$€ØNSED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***. SEE DOCUMENTATIØNIPA€MAGE:204-9Ø5M99.8/      3 V
115  00062AI          GO TO 900                                                                                    115
116                                                                                                               116
117  000630I   115    CONTINUE                                                                                     117
118  000630I          XOMEGA=0.9                                                                                   118
119  00063CI          DO 120 I=1,M                                                                                 119
120  000650I          P(I,1)=P(I,1)+XOMEGA*(P(MP,NP)-P(I,NP))                                                      120
121  0006A6I   120    CONTINUE                                                                                     121
122  00068EI          GO TO 50                                                                                     122
123               C                                                                                               123
124               C                                                                                               124
125               C     STANDARD PRINTOUT                                                                         125
126               C                                                                                               126
127               C                                                                                               127
128  0006C2I   900    CONTINUE                                                                                     128
129  0006C2I          DO 920 J=1,NP                                                                                129
130  0006D6I          CALL SPRINT(J)                                                                               130
131  0006ECI   920    CONTINUE                                                                                     131
132  000704I          GO TO 980                                                                                    132
133               C                                                                                               133
134               C                                                                                               134
135               C     ERROR PRINTOUT                                                                            135
136               C                                                                                               136
137               C                                                                                               137
138  00070AI   950    CONTINUE                                                                                     138
139  00070AI          WRITE(IPRINT,960) MAXITP                                                                     139
140  000728I   960    FORMAT(////,'  NO CONVERGENCE IN',I6,' OVERALL                                               140
141                 1               ITERATIONS ON P',///)                                                         141
142                                                                                                               142
143  00078AI          CALL SPRINT(NP)                                                                              143
144  0007A0I   980    CONTINUE                                                                                     144
145                                                                                                               145
146  0007A0I          WRITE (8)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                146
147  0007DCI          WRITE (8)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                147
148  000810I          DO 990  J = 1, NP                                                                            148
149  000820I          WRITE (8)  (U(I,J), I = 1, MP)                                                               149
150  0008ACI   990    CONTINUE                                                                                     150
151  0008C4I          DO 991  J = 1, NP                                                                            151
152  0008D8I          WRITE (8)  (V(I,J), I = 1, MP)                                                               152
153  000960I   991    CONTINUE                                                                                     153
154  000978I          WRITE (9)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                154
155  0009B4I          WRITE (9)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                155
156  0009E4I          DO 992  J = 1, NP                                                                            156
157  0009F8I          WRITE (9)  (T(I,J), I = 1, MP)                                                               157
158  000A80I   992    CONTINUE                                                                                     158
159  000A98I          DO 993  J = 1, NP                                                                            159
160  000AACI          WRITE (9)  (P(I,J), I = 1, MP)                                                               160
161  000B34I   993    CONTINUE                                                                                     161
162  000B4CI          WRITE (7)  A, SIGMA, XNUIN, XKIN, YMAX, ETLINE, ETOLP, DX, DY                                162
163  000B88I          WRITE (7)  M, N, MAXITL, MAXITP, KOUT, IGUESS                                                163
164  000BB8I          DO 994  J = 1, NP                                                                            164
165  000BCCI          WRITE (7)  (RHO(I,J), I = 1, MP)                                                             165
166  000C54I   994    CONTINUE                                                                                     166
167  000C6CI          STOP                                                                                         167
168  000C74I          END                                                                                          168

NO  ERRORS:F7D  R04-00     MAINPROG    FIRE2     07/17/81  10:17:32  TABLE SPACE:   4 KB
STATEMENT BUFFER:   20 LINES/1321 BYTES    STACK SPACE: 175 WORDS
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Table C.3--Continued

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FORTRAN-VIID:RO4GBRSED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATION PACKAGE 284-R05R99.V/      5 V
000000I          SUBROUTINE HCFUNC(KEY,II,JJ)


000004I          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),
                1        X(41),Y(241),
                2        A,SIGMA,XNUIN,XKIN,YMAX,M,N,
                3        MP,NP,DX,DY,DYDX,DXDY,NNN,
                4        ETLINE,MAXITL,ETOLP,MAXITP,
                5        QVAL,XNU,XK,QPRIM,
                6        KOUT, IGUESS,
                7        IPRINT,ITTY


000004I          GOTO(100,200,300,400,500)KEY
          C
          C
          C      U INLET PRESCRIPTION - CAN BE CHANGED
          C
          C
000036I   100    CONTINUE
000036I          READ(ITTY,90) CCCIN
000054I   90     FORMAT(F10.0)
          C
000060I          CONTINUE
000060I          DO 150 J=1,NP
000074I          CC=1.0
000080I          CCC=CCCIN
0000BCI          CFAC=(EXP(CC)-EXP(-CC))/(EXP(CC)+EXP(-CC))
000132I          U(MP,J)=-CCC*CFAC*EXP(-CCC*Y(J))
00019CI          U(MP,J)=-CCC
0001C2I          IF(Y(J).GT.1.0) U(MP,J)=-CCC*EXP(-(Y(J)-1.0))
00023CI   150    CONTINUE
          C
          C
          C           INITIAL GUESSES FOR Y=0.0 - CAN BE CHANGED
          C
          C
000254I          IF (IGUESS .EQ. 0)  GO TO 170
00026AI          DO 160  I = 1, MP
00027EI          READ (ITTY,161)  P(I,1)
0002B0I   160    CONTINUE
0002C8I   161    FORMAT (F10.0)
0002D4I          GO TO 170

0002DAI   170    CONTINUE
0002DAI          DO 180  I = 1, MP
0002EEI          P(I,1) = 0.67-4.0+4.0*X(I)**2-0.67*X(I)**3+1.0
000356I          P(I,1)=X(I)*X(I)-1.0
000386I          T(I,1) = 1.6-0.6*X(I)
0003B0I          CC = 1.
0003BCI          CCC=CCCIN
0003C8I          EP = EXP(CC*X(I))
0003FEI          EM = EXP(-CC*X(I))
00043EI          U(I,1) = -CCC*(EP-EM)/(EP+EM)
000474I          U(I,1)=U(I,1)/CFAC

000496I    180   CONTINUE
```

Table C.3--Continued

```
FORTRAN-VIID:R04.00 RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATION PAGE 204-R04.00/      6 V
 58  0004AEI          GOTO 900                                                                              226
 59            C                                                                                            227
 60            C                                                                                            228
 61            C     Q(X,Y) PRESCRIPTION - CAN BE CHANGED                                                   229
 62            C                                                                                            230
 63            C                                                                                            231
 64  0004B4I   200    CONTINUE                                                                              232
 65  0004B4I          IF(JJ.EQ.1)TAVG=T(II,JJ)                                                              233
 66  0004E4I          JJM=JJ-1                                                                              234
 67  0004F2I          IF(JJ.GE.2)TAVG=0.5*(T(II,JJ)+T(II,JJM))                                              235
 68                                                                                                         236
 69  000542I          QVAL=1.0-SIGMA*(TAVG**4-1.0)                                                          237
 70                                                                                                         238
 71  00056AI          IF(Y(JJ).GT.1.0) QVAL=QVAL-1.0                                                        239
 72            C                                                                                            240
 73            C                                                                                            241
 74            C     NU(X,Y) PRESCRIPTION - CAN BE CHANGED                                                  242
 75            C                                                                                            243
 76            C                                                                                            244
 77  000596I   300    CONTINUE                                                                              245
 78  000596I          XNU = XNUIN                                                                           246
 79  0005A2I          CNU=0.0                                                                               247
 80  0005AEI          XNU=XNU*EXP(-CNU*X(II))                                                               248
 81  0005F4I          IF(X(II).GT.1.0)XNU=0.0                                                               249
 82  00061AI          GOTO 900                                                                              250
 83            C                                                                                            251
 84            C                                                                                            252
 85            C     K(X,Y) PRESCRIPTION - CAN BE CHANGED                                                   253
 86            C                                                                                            254
 87            C                                                                                            255
 88  000520I   400    CONTINUE                                                                              256
 89  000520I          XK = XKIN                                                                             257
 90  00052CI          CK=0.0                                                                                258
 91  000538I          XK=X<*EXP(-CK*X(II))                                                                  259
 92  00057CI          IF(X(II).GT.1.0)XK=0.0                                                                260
 93  0006A2I          GOTO 900                                                                              261
 94            C                                                                                            262
 95            C     QPRIME COMPUTATION - CAN BE CHANGED                                                    263
 96            C                                                                                            264
 97  0006A8I   500    CONTINUE                                                                              265
 98  0006A8I          IF(JJ.EQ.1)TAVG=T(II,JJ)                                                              266
 99  0006D8I          JJM=JJ-1                                                                              267
100  0006E6I          IF(JJ.GE.2)TAVG=(T(II,JJ)+T(II,JJM))/2.                                               268
101                                                                                                         269
102  000736I          QPRIM=-4.*SIGMA*(TAVG**3)                                                             270
103                                                                                                         271
104  000760I   900    CONTINUE                                                                              272
105  000760I          RETURN                                                                                273
106  000766I          END                                                                                   274

NO  ERRORS:F7D  R04-00     SUBROUTINE  RCFUNC    07/17/81  10:17:36  TABLE SPACE:   4 KB
STATEMENT BUFFER:   20 LINES/1321 BYTES    STACK SPACE: 213 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
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Table C.3--Continued

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FORTRAN-VIID:RQ$6@@SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMEN@TATI@@1PA@@@@@:284-@@$@99.¥/      7 V
  1  000000I          SUBROUTINE SPRINT(JJ)                                                              275
  2                                                                                                      276
  3                                                                                                      277
  4  000004I          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                       278
  5                  1       X(41),Y(241),                                                              279
  6                  2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                               280
  7                  3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                 281
  8                  4       ETLINE,MAXITL,ETOLP,MAXITP,                                                282
  9                  5       QVAL,XNU,XK,QPRIM,                                                         283
 10                  6       KOUT, IGUESS,                                                              284
 11                  7       IPRINT,ITTY                                                                285
 12                                                                                                      286
 13  000004I          DIMENSION  PX(41), VISC(41), DIFF(41), HEAT(41)                                    287
 14            C                                                                                         288
 15            C                                                                                         289
 16            C     COMPUTE PX, UXX, TXX AND Q(T) TERMS                                                 290
 17            C                                                                                         291
 18            C                                                                                         292
 19  000004I          DO 50  I = 1, MP                                                                   293
 20  000280I          IP = I + 1                                                                         294
 21  000288I          IM = I - 1                                                                         295
 22  0002CCI          DO 10  KK = 2, 4                                                                   296
 23  0002D4I          CALL BCFUNC (KK, I, JJ)                                                            297
 24  0002F4I    10    CONTINUE                                                                           298
 25  000304I          IF (I .EQ. 1)  GO TO 20                                                            299
 26  000320I          IF (I .EQ. MP)  GO TO 30                                                           300
 27                                                                                                      301
 28  00033BI          PX(I) = (P(IP,JJ)-P(IM,JJ))/(2.*DX)                                               302
 29  000386I          VISC(I) = (U(IP,JJ)-2.*U(I,JJ)+U(IM,JJ))*XNU/(DX*DX)                              303
 30  0003FCI          DIFF(I) = (T(IP,JJ)-2.*T(I,JJ)+T(IM,JJ))*XK/(DX*DX)                               304
 31  000472I          HEAT(I) = QVAL                                                                     305
 32  000484I          GO TO 50                                                                           306
 33                                                                                                      307
 34  00048AI    20    PX(I) = 0.                                                                         308
 35  00049CI          VISC(I) = 0.                                                                       309
 36  0004AEI          DIFF(I) = 2.*(T(IP,JJ)-T(I,JJ))*XK/(DX*DX)                                        310
 37  0004C8I          HEAT(I) = QVAL                                                                     311
 38  0004DAI          GO TO 50                                                                           312
 39                                                                                                      313
 40  000520I    30    PX(I) = (P(I,JJ)-P(IM,JJ))/DX                                                      314
 41  000546I          VISC(I) = VISC(IM)                                                                 315
 42  00057EI          DIFF(I) = DIFF(IM)                                                                 316
 43  000596I          HEAT(I) = QVAL                                                                     317
 44                                                                                                      318
 45  0005A8I    50    CONTINUE                                                                           319
 46            C                                                                                         320
 47            C                                                                                         321
 48            C     STANDARD PRINTOUT                                                                   322
 49            C                                                                                         323
 50            C                                                                                         324
 51  0005C0I          WRITE(IPRINT,100)JJ,Y(JJ)                                                          325
 52  0005E8I    100   FORMAT(//// ' FOR J =' I5 '  (Y =' F8.3 '):')                                     326
 53  000620I          WRITE(IPRINT,120)                                                                  327
 54  000643CI   120   FORMAT(//,'          I      X        U        V        T',                        328
 55                 1    '       RHO      P' 21X 'PX      NU*UXX    K*TXX    Q(T)' /)                329
 56  0006A2I          DO 150 I=1,MP                                                                      330
 57  0006B6I          WRITE(IPRINT,140)I,X(I),U(I,JJ),V(I,JJ),T(I,JJ),RHO(I,JJ),                        331
```

Table C.3--Continued

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FORTRAN-VIID:R04-00BASED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATION PAGE 2-204-FOR 899.0/      8 V
 58                    1                P(I,JJ), PX(I), VISC(I), DIFF(I), HEAT(I)                              332
 59  000804I    140    FORMAT(3X,I6,6F8.3, 15X, 4F8.3)                                                         333
 60  00081CI    160    CONTINUE                                                                                334
 61  0008341           RETURN                                                                                  335
 62  000834I           END                                                                                     336

NO  ERRORS:F7D  R04-00     SUBROUTINE  SPRINT    07/17/81  10:17:39  TABLE SPACE:    3 KB
STATEMENT BUFFER:    20 LINES/1321 BYTES    STACK SPACE: 171 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
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Table C.3--Continued

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FORTRAN-VIID:RO[illegible]SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336        ***, SEE DOCUMENTATION IPA[illegible]  9 V
  1  000000I          SUBROUTINE MSWEEP                                                                    337
  2                                                                                                       338
  3                                                                                                       339
  4  000004I          COMMON U(41,241),V(41,241),P(41,241),T(41,241),RHO(41,241),                          340
  5                  1       X(41),Y(241),                                                                341
  6                  2       A,SIGMA,XNUIN,XKIN,YMAX,M,N,                                                 342
  7                  3       MP,NP,DX,DY,DYDX,DXDY,NNN,                                                   343
  8                  4       ETLINE,MAXITL,ETOLP,MAXITP,                                                  344
  9                  5       QVAL,XNU,XK,QPRIM,                                                           345
 10                  6       KOUT, IGUESS,                                                                346
 11                  7       IPRINT,ITTY                                                                  347
 12                                                                                                       348
 13  0000041          DIMENSION XX(200),FF(200),AZ(100),A1(100),                                           349
 14                  1          A2(100),BZ(100),B1(100),B2(100),                                          350
 15                  2          CZ(100),C1(100),C2(100),DZ(100),                                          351
 16                  3          D1(100),D2(100),XMAT(7,200),                                              352
 17                  4          DNEW(200)                                                                 353
 18                                                                                                       354
 19                                                                                                       355
 20  000004I          MM=M-1                                                                               356
 21  00321AI          MB=2*(M-1)                                                                           357
 22  00322EI          MBM = MB - 1                                                                         358
 23  00323CI          MBM2 = MB - 2                                                                        359
 24  00324AI          MBM3 = MB - 3                                                                        360
 25                                                                                                       361
 26                                                                                                       362
 27  003258I          DO 800 J=1,NP                                                                        363
 28  00326CI          JM=J-1                                                                               364
 29  00327AI          JM2=J-2                                                                              365
 30             C                                                                                         366
 31             C                                                                                         367
 32             C     INITIAL GUESSES FOR U AND T ON LINE J                                               368
 33             C                                                                                         369
 34             C                                                                                         370
 35  003288I          DO 20 I=2,M                                                                          371
 36  00329CI          IU=2*(I-1)-1                                                                         372
 37  0032B2I          IT=2*(I-1)                                                                           373
 38  0032C6I          IF(J.EQ.1) GO TO 14                                                                  374
 39  0032DCI          IF(J .EQ. 2) GO TO 12                                                                375
 40             C                                                                                         376
 41             C     LINEAR EXTRAPOLATION USED FOR J .GE. 3                                              377
 42             C                                                                                         378
 43  0032F2I          U(I,J)=2.*U(I,JM)-U(I,JM2)                                                           379
 44  003346I          T(I,J)=2.*T(I,JM)-T(I,JM2)                                                           380
 45  00339AI          GO TO 14                                                                             381
 46             C                                                                                         382
 47             C     PREVIOUS VALUES (=INITIAL GUESSES FOR NNN=1) USED FOR J = 1 OR J = 2                383
 48             C                                                                                         384
 49  0033A0I     12   U(I,J)=U(I,JM)                                                                       385
 50  0033D4I          T(I,J)=T(I,JM)                                                                       386
 51  003408I     14   CONTINUE                                                                             387
 52  00340BI          XX(IU)=U(I,J)                                                                        388
 53  00342FI          XX(IT)=T(I,J)                                                                        389
 54  003454I          IF(I.EQ.2)T(1,J)=T(I,J)                                                              390
 55  003492I     20   CONTINUE                                                                             391
 56             C                                                                                         392
 57             C                                                                                         393
```

Table C.3--Continued


```
                   C
                   C      OVERALL VECTOR NEWTON ITERATION
                   C      USING GAUSS ELIMINATION ON LINEAR EQNS.
                   C
                   C
                   C
         0034AAI          NUMITL = 0
         0034B2I   100    NUMITL=NUMITL+1
         0034C0I          IF(NUMITL.GT.MAXITL)GOTO 900


         0034D8I          DO 200 I=2,M
         0034ECI          IM=I-1
         0034FAI          IP=I+1
         003508I          IU=2*(I-1)-1
         00351EI          IT=2*(I-1)
         003532I          IF(J.EQ.1) JM=1

         00354AI          KK=2
         003552I          CALL BCFUNC(KK,I,J)
         003570I          KK=3
         003578I          CALL BCFUNC(KK,I,JM)
         003598I          XNUM=XNU
         0035A4I          KK=3
         0035ACI          CALL BCFUNC(KK,I,J)
         0035CCI          KK=4
         0035D4I          CALL BCFUNC(KK,I,JM)
         0035F4I          XKM=XK
         003600I          KK=4
         003608I          CALL BCFUNC(KK,I,J)
         003628I          KK=5
         003630I          CALL BCFUNC(KK,I,J)

         003650I          IF(J.GE.2) GO TO 110
                   C
                   C
                   C      NEWTON SET-UP FOR J = 1
                   C
                   C
         003666I          RR=0.5*(P(IP,1)-P(IM,1))
         00368EI          PI=0.

         00369AI          FF(IU)=(U(IP,1)-2.*U(I,1)+U(IM,1))*XNU/DX
                        1        -0.5*(U(IP,1)-U(IM,1))*U(I,1)/T(I,1)-RR

         00372AI          FF(IT)=(T(IP,1)-2.*T(I,1)+T(IM,1))*XK/DX
                        1        -0.5*(T(IP,1)-T(IM,1))*U(I,1)/T(I,1)+DX*QVAL

         0037C2I          IF(IM.GE.2) GO TO 108
         0037D8I          AZ(1)=0.0
         0037F2I          A1(1)=-2.*XNU/DX-0.5*U(3,1)/T(2,1)
         003814I          A2(1)=XNU/DX-0.5*U(2,1)/T(2,1)
         003838I          BZ(1)=0.0
         003842I          B1(1)=0.5*U(3,1)*U(2,1)/(T(2,1)**2)
         00386AI          B2(1)=0.0
         003872I          CZ(1)=0.0
         00387CI          C1(1)=-0.5*(T(3,1)/T(2,1)-1.0)
```

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FORTRAN-VIID:R04.00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336      ***, SEE DOCUMENTATION PACKAGE 4284-000099.B/     11 V
115  0038A0I          C2(1)=0.0                                                                                    451
116  0038AAI          DZ(1)=0.0                                                                                    452
117  0038B4I          D1(1)=-XK/DX+0.5*T(3,1)*U(2,1)/(T(2,1)**2)                                                   453
118                  1        *DX*QPRIM                                                                           454
119  0038FEI          D2(1)=XK/DX-0.5*U(2,1)/T(2,1)                                                                455
120  003922I          GO TO 200                                                                                    456
121                                                                                                               457
122  003928I  108     AZ(IM)=XNU/DX+0.5*U(I,1)/T(I,1)                                                              458
123  003964I          A1(IM)=-2.*XNU/DX-0.5*(U(IP,1)-U(IM,1))/T(I,1)                                               459
124  00398CI          A2(IM)=XNU/DX-0.5*U(I,1)/T(I,1)                                                              460
125  0039F8I          BZ(IM)=0.0                                                                                   461
126  003A0AI          B1(IM)=0.5*(U(IP,1)-U(IM,1))*U(I,1)/(T(I,1)**2)                                              462
127  003A6AI          B2(IM)=0.0                                                                                   463
128  003A7CI          C2(IM)=0.0                                                                                   464
129  003A8EI          C1(IM)=-0.5*(T(IP,1)-T(IM,1))/T(I,1)                                                         465
130  003AD2I          C2(IM)=0.0                                                                                   466
131  003AE4I          DZ(IM)=XK/DX+0.5*U(I,1)/T(I,1)                                                               467
132  003920I          D1(IM)=-2.*XK/DX+0.5*(T(IP,1)-T(IM,1))*U(I,1)                                                468
133                  1        /(T(I,1)**2)+DX*QPRIM                                                               469
134  003BAAI          D2(IM)=XK/DX-0.5*U(I,1)/T(I,1)                                                               470
135                                                                                                               471
136  003BE6I          IF(I.LT.M) GO TO 200                                                                         472
137  003BFEI          A2(IM)=0.0                                                                                   473
138  003C10I          D2(IM)=0.0                                                                                   474
139                                                                                                               475
140  003C22I          GO TO 200                                                                                    476
141              C                                                                                                477
142              C                                                                                                478
143              C     NEWTON SET-UP FOR J .GE. 2                                                                 479
144              C                                                                                                480
145              C                                                                                                481
146  003C28I  110    CONTINUE                                                                                     482
147  003C28I         RR=(U(IP,JM)-2.*U(I,JM)+U(IM,JM))*XNUM                                                       483
148                 1   /(2.*DX)-((U(IP,JM)**2)/T(IP,JM)                                                         484
149                 1   -(U(IM,JM)**2)/T(IM,JM)+2.*(P(IP,JM)                                                     485
150                 3   -P(IM,JM))-0.5*A*DY*(1./T(IP,JM)                                                         486
151                 4   -1./T(IM,JM)))/4.                                                                        487
152  003D8AI         RR=-RR-U(I,JM)*V(I,JM)/T(I,JM)*DXDY                                                          488
153  003E22I         PI=-((T(IP,JM)-2.*T(I,JM)+T(IM,JM))*XKM                                                      489
154                 1   /(2.*DX)-(U(IP,JM)-U(IM,JM))/4.+V(I,JM)*DXDY)                                            490
155                                                                                                               491
156  003EF8I         FF(IJ)=(U(IP,J)-2.*U(I,J)+U(IM,J))*XNU                                                       492
157                 1       /(2.*DX)-0.25*((U(IP,J)**2)/T(IP,J)                                                  493
158                 2       -(U(IM,J)**2)/T(IM,J)-0.5*A*DY                                                       494
159                 3       *(1./T(IP,J)-1./T(IM,J)))-U(I,J)*                                                    495
160                 4       (DXDY*V(I,JM)/T(I,JM)-0.25*(U(IP,J)                                                  496
161                 5       /T(IP,J)+U(IP,JM)/T(IP,JM)-U(IM,J)                                                   497
162                 6       /T(IM,J)-U(IM,JM)/T(IM,JM)))-RR                                                      498
163                                                                                                               499
164  00419AI         FF(IT)=((T(IP,J)-2.*T(I,J)+T(IM,J))*XK                                                       500
165                 1       /(2.*DX)-0.25*(U(IP,J)-U(IM,J))                                                      501
166                 2       -T(I,J)*(DXDY*V(I,JM)/T(I,JM)-0.25                                                   502
167                 3       *(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)                                                  503
168                 4       -U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))                                                 504
169                 5       +DX*QVAL-PI                                                                          505
170                                                                                                               506
171  0043A6I         IF(IM.GE.2)GOTO 120                                                                          507
```

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FORTRAN-VIID:RQ$E00SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUME@TATI0@1PA[84GZ:264-205899.V/     12 V
172  0043AC1          AZ(I)=0.0                                                                                    508
173  0043B61          A1(I)=-2.*XNU/(2.*DX)-(DXDY*V(2,JM)/T(2,JM)                                                  509
174                  1       -0.25*(U(3,J)/T(3,J)+U(3,JM)/T(3,JM))                                                510
175  0044661          A2(I)=XNU/(2.*DX)-0.25*(2.*U(3,J)-U(2,J))/T(3,J)                                             511
176  0044C81          BZ(I)=0.0                                                                                    512
177  0044D21          B1(I)=0.25*(0.5*A*DY/(T(2,J)**2))                                                            513
178  00450C1          B2(I)=0.25*((U(3,J)**2-U(2,J)*U(3,J))/(T(3,J)                                                514
179                  1       **2)-0.5*A*DY/(T(3,J)**2))                                                           515
180  0045B81          CZ(I)=0.0                                                                                    516
181  0045C21          C1(I)=0.0                                                                                    517
182  0045CC1          C2(I)=-0.25*(1.-T(2,J)/T(3,J))                                                               518
183  00460E1          DZ(I)=0.0                                                                                    519
184  0046181          D1(I)=-XK/(2.*DX)-(DXDY*V(2,JM)/T(2,JM)                                                      520
185                  1       -0.25*(U(3,J)/T(3,J)+U(3,JM)/T(3,JM))                                                521
186                  2       +0.5*DX*QPRIM                                                                        522
187  0046D61          D2(I)=XK/(2.*DX)-0.25*T(2,J)*U(3,J)/(T(3,J)**2)                                              523
188  00473C1          GOTO 200                                                                                     524
189                                                                                                               525
190  0047421  120    AZ(IM)=XNU/(2.*DX)+0.25*(2.*U(IM,J)-U(I,J))/T(IM,J)                                           526
191  0047C01          A1(IM)=-2.*XNU/(2.*DX)-(DXDY*V(I,JM)/T(I,JM)                                                 527
192                  1       -0.25*(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)                                             528
193                  2       -(U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))                                                529
194  00490A1          A2(IM)=XNU/(2.*DX)-0.25*(2.*U(IP,J)-U(I,J))/T(IP,J)                                          530
195  0049881          BZ(IM)=-0.25*((U(IM,J)-U(I,J))*U(IM,J)/(T(IM,J)                                              531
196                  1       **2)-0.5*A*DY/(T(IM,J)**2))                                                          532
197  004A581          B1(IM)=0.0                                                                                   533
198  004A6A1          B2(IM)=0.25*((U(IP,J)-U(I,J))*U(IP,J)/(T(IP,J)                                               534
199                  1       **2)-0.5*A*DY/(T(IP,J)**2))                                                          535
200  004B321          CZ(IM)=0.25*(1.-T(I,J)/T(IM,J))                                                              536
201  004B801          C1(IM)=0.0                                                                                   537
202  004B921          C2(IM)=-0.25*(1.-T(I,J)/T(IP,J))                                                             538
203  004BE81          DZ(IM)=XK/(2.*DX)+0.25*T(I,J)*U(IM,J)/(T(IM,J)**2)                                           539
204  004C741          D1(IM)=-2.*XK/(2.*DX)-(DXDY*V(I,JM)/T(I,JM)                                                  540
205                  1       -0.25*(U(IP,J)/T(IP,J)+U(IP,JM)/T(IP,JM)                                             541
206                  2       -U(IM,J)/T(IM,J)-U(IM,JM)/T(IM,JM)))                                                 542
207                  3       +0.5*DX*QPRIM                                                                        543
208  004D001          D2(IM)=XK/(2.*DX)-0.25*T(I,J)*U(IP,J)/(T(IP,J)**2)                                           544
209                                                                                                               545
210  004E5C1          IF(I.LT.M)GOTO 200                                                                           546
211  004E741          A2(IM)=0.0                                                                                   547
212  004E881          B2(IM)=0.0                                                                                   548
213  004E9C1          C2(IM)=0.0                                                                                   549
214  004EAE1          D2(IM)=0.0                                                                                   550
215                                                                                                               551
216  004EC01  200    CONTINUE                                                                                     552
217              C                                                                                                553
218              C                                                                                                554
219              C      SPARSE (JACOBIAN) MATRIX COMPACTIFICATION                                                 555
220              C                                                                                                556
221              C                                                                                                557
222  004ED81          DO 250 IM=1,MM                                                                               558
223  004EEC1          IU=2*IM-1                                                                                    559
224  004EFC1          IT=2*IM                                                                                      560
225  004F0A1          XMAT(1,IU)=0.0                                                                               561
226  004F221          XMAT(2,IU)=AZ(IM)                                                                            562
227  004F421          XMAT(3,IU)=BZ(IM)                                                                            563
228  004F621          XMAT(4,IU)=A1(IM)                                                                            564
```

Table C.3--Cᵔntinued


```
004F82I         XMAT(5,IU)=B1(IM)
004FA2I         XMAT(6.IU)=A2(IM)
004FC2I         XMAT(7,IU)=B2(IM)
004FE2I         XMAT(1.IT)=CZ(IM)
005002I         XMAT(2.IT)=DZ(IM)
005020I         XMAT(3.IT)=C1(IM)
00503FI         XMAT(4.IT)=D1(IM)
00505CI         XMAT(5,IT)=C2(IM)
00507AI         XMAT(6,IT)=D2(IM)
005098I         XMAT(7.IT)=0.0
0050B0I   250   CONTINUE
          C
          C
          C     GAUSS ELIMINATION TO SOLVE JD=-F
          C
          C
0050C8I         DO 500 I=1,MBM
0050DCI         KMAX=3
0050E4I         IF(I.EQ.MBM2) KMAX=2
0050FEI         IF(I.EQ.MBM) KMAX=1

005118I         DO 490 K=1,KMAX
00512CI         IK=I+K
00513EI         M4K=4-K
00514CI         PIVOT=XMAT(M4K.IK)/XMAT(4.I)

00517CI         DO 480 LL=4,7
005184I         ML=LL-K
005196I         XMAT(ML.IK)=XMAT(ML.IK)-PIVOT*XMAT(LL.I)
0051E6I   480   CONTINUE

0051FCI         FF(IK)=FF(IK)-PIVOT*FF(I)
00522EI   490   CONTINUE
005246I   500   CONTINUE

00525EI         DNEW(MB)=-FF(MB)/XMAT(4,MB)
005292I         DNEW(MBM)=-(FF(MBM)+XMAT(5,MBM)*DNEW(MB))/XMAT(4.MBM)
0052E4I         DNEW(MBM2)=-(FF(MBM2)+XMAT(5,MBM2)*DNEW(MBM)
               1              +XMAT(6,MBM2)*DNEW(MB))/XMAT(4,MBM2)

005356I         DO 550 I=1.MBM3
00536AI         IB=MBM2-I
00537CI         XXX=FF(IB)
005390I         DO 540 K=5,7
005398I         IBP=IB+K-4
0053ACI         XXX=XXX+XMAT(K,IB)*DNEW(IBP)
0053DCI   540   CONTINUE
0053F2I         DNEW(IB)=-XXX/XMAT(4,IB)
00541EI   550   CONTINUE
          C
          C
          C     CHECK FOR NEWTON CONVERGENCE
          C
          C
005436I         IFLAG=0
00543EI         DO 420 IB=1.MB
005452I         XX(IB)=XX(IB)+DNEW(IB)
```

Table C.3--Continued

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FORTRAN-VIID:RQ&E@@SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUME@TATI@@1PAC@AG@:284-P03899.@/     14 V
286  00547AI          IF(ABS(ONEW(IB)).GT.ETLINE)IFLAG=1                                    622
287  005488I   420    CONTINUE                                                              623
288                                                                                         624
289  0054D0I          DO 450 I=2,M                                                          625
290  0054E4I          IU=2*(I-1)-1                                                          626
291  0054FAI          IT=2*(I-1)                                                            627
292  00550FI          U(I,J)=XX(IU)                                                         628
293  005536I          T(I,J)=XX(IT)                                                         629
294  00555EI          IF(I.EQ.2)T(1,J)=T(I,J)                                               630
295  00559CI   450    CONTINUE                                                              631
296                                                                                         632
297  0055B4I          IF(IFLAG.EQ.0)GOTO 700                                                633
298  0055CAI          GOTO 100                                                              634
299               C                                                                         635
300               C                                                                         636
301               C     SUCCESSFUL LINE SWEEP TERMINATION                                   637
302               C                                                                         638
303               C                                                                         639
304  0055CEI   700    CONTINUE                                                              640
305                                                                                         641
306  0055CEI          DO 750 I=1,MP                                                         642
307                                                                                         643
308  0055E2I          IF (J .EQ. 1)  GO TO 740                                              644
309                                                                                         645
310  0055F8I          IP=I+1                                                                646
311  005606I          IF(I.EQ.MP)IP=I                                                       647
312  005624I          IM=I-1                                                                648
313  005632I          IF(I.EQ.1)IM=I                                                        649
314  00564EI          FRAC=0.25                                                             650
315  00565AI          IF(I.EQ.1)FRAC=0.5                                                    651
316  005676I          IF(I.EQ.MP)FRAC=0.5                                                   652
317                                                                                         653
318  005694I          V(I,J)=V(I,JM)*T(I,J)/T(I,JM)-FRAC*DYDX                               654
319                  1       *T(I,J)*(U(IP,J)/T(IP,J)+U(IP,JM)                             655
320                  2       /T(IP,JM)-U(IM,J)/T(IM,J)-U(IM,JM)                            656
321                  3       /T(IM,JM))                                                    657
322                                                                                         658
323  0057FCI          P(I,J)=P(I,JM)-0.5*A*DY*(1./T(I,J)+1./T(I,JM))                        659
324  005888I   740    RHO(I,J)=1./T(I,J)                                                    660
325                                                                                         661
326  005AC2I   750    CONTINUE                                                              662
327               C                                                                         663
328               C                                                                         664
329               C     IF(J.EQ.1) GOTO 980                                                 665
330               C                                                                         666
331               C                                                                         667
332  005ADAI   800    CONTINUE                                                              668
333  0054F2I          GOTO 980                                                              669
334               C                                                                         670
335               C                                                                         671
336               C     ERROR PRINTOUT - NO CONVERGENCE                                     672
337               C                                                                         673
338               C                                                                         674
339  005AFAI   900    CONTINUE                                                              675
340  0059FAI          WRITE(IPRINT,905)  J, NNN, MAXITL                                     676
341  005920I - 905    FORMAT(///,'  NO NEWTON CONVERGENCE ON LINE',                         677
342                  1       I6 ' OF OVERALL SWEEP' I6 ' IN' I6 ' ITERATIONS')            678
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Table C.3--Continued

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FORTRAN-VIID:RD46B0SED RESTRICTED RIGHTS AS STATED IN LICENSE L-0336          ***, SEE DOCUMENTATION/RIPA[illegible]284-283B99.97/     15 V
343  0059781          KOUT=0                                                                    679
344  0059801          DO 920 JA=1,J                                                             680
345  0059941          CALL SPRINT(JA)                                                           681
346  0059ACI   920    CONTINUE                                                                  682
347                                                                                             683
348  0059C4I   980    CONTINUE                                                                  684
349  0059C4I          RETURN                                                                    685
350  0059CCI          END                                                                       686

NO  ERRORS:F7D  R04-00     SUBROUTINE  MSWEEP    07/17/81  10:18:01  TABLE SPACE:   6 KB
STATEMENT BUFFER:   20 LINES/1321 BYTES    STACK SPACE: 228 WORDS
SINGLE PRECISION FLOATING PT SUPPORT REQUIRED FOR EXECUTION
```

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EXECUTIVE SUMMARY

Large urban fires have resulted from natural disasters, explosions, and wartime actions. In many cases, entire urban areas were totally destroyed despite firefighters' efforts to contain the flames. The World War II firebombing raids on European and Japanese population centers caused immense damage and hundreds of thousands of casualties. Several ignited firestorms, with hurricane-force winds, high street-level temperatures, high concentrations of carbon monoxide, and complete burning of all combustible materials within the firestorm boundaries. Firestorms also produced a high number of casualties, seldom ameliorated even by concerted rescue efforts.

Large urban fires are a much greater threat in the age of nuclear weapons than ever before. Hundreds of square kilometers of an urban (or wildland) area can be ignited simultaneously by a single-megaton nuclear weapon. Indeed, superfires of unprecedented size could dwarf the tremendous fires of World War II.

After the firestorm in Hamburg, civil defense leaders in Germany addressed the problem of protecting the cities and the civilian population from such fierce fires. On the basis of the meager technical information then available, guidelines for rescue operations were outlined. However, the physical characteristics of the firestorm environment had not yet been defined--e.g., winds, temperatures, gas concentrations, sizes of combustible areas, and populations at risk. Such information is essential if planners are to design procedures for fire suppression/containment and population protection/rescue.

Today, years later, we are still without a complete and accurate picture of the superfire environment. No analysis has ever focused on the nature of the flow in and around the fire itself--which is the most vital information. Most studies have simply attempted to extrapolate results from classical plume theories, known to be valid only well above a small free-burning fire.

This report uses a new analytical approach to analyze the superfire environment. Our approach is based on a "component" model in

which different sets of equations are employed to represent the fundamentally different physics in separate parts of the fire-generated flow. Asymptotic analysis is used to derive the individual equation sets from standard conservation laws.

The "component" approach allows us to consider in detail the nature of the superfire environment in the critical region in and around the fire itself. Our analysis of that "combustion" region constitutes the first--and most interesting--model component.

The intensity and extent of the fires in the combustion region largely determine the characteristics of the entire flow field. Cold air is drawn into the burning zone and heated by the ongoing combustion processes. The heated air rises buoyantly, establishing a natural convection column. The buoyant air in the column cools and expands until a local equilibrium is established either in the upper atmosphere or at an inversion layer (of sufficient magnitude). Finally, a mesoscale recirculation pattern is established similar to a nocturnal urban heat island flow. Analyses of the convection column and recirculation flows produce the other basic model components.

We focus on the combustion region. Our analysis is valid in the upper limit for large urban fires with horizontal dimensions on the order of tens of kilometers and in the lower limit for fires the size of the multiple-fuel-bed Flambeau fires.

The first predictions concern the environment for a nominal large urban fire (with a 10 km radius). The sensitivity of the predictions to changes in model parameters is then considered. An experimental Flambeau fire (of radius $\sim$0.25 km) is also analyzed. The results show reasonable agreement in all parameters when inflow horizontal velocities are selected to correspond to those observed.

Numerical simulation also shows that the type of plume observed over the Flambeau fire (i.e., with a width comparable to that of the fire) can be established only if turbulent mixing plays a major role in the combustion region dynamics. We anticipate that turbulence will play such a role in large urban fires.